Performance Improvement via Iterative Connection of Passive Systems

Kengo Urata, Masaki Inoue *Member, IEEE,* Takayuki Ishizaki *Member, IEEE,* and Jun-ichi Imura *Member, IEEE.*

***Abstract*—This paper addresses *model-set-based* quantitative analysis of feedback systems. In particular, we find a model set describing the subsystems such that the performance improvement of the feedback system is achieved. To this end, we introduce the parameter-integrated passivity to accurately describe each passive subsystem and their feedback system; a model set describing passive systems is characterized by the two matrix parameters. The matrix parameters enable to evaluate the L_2-gain "of the model set", which is defined as the L_2-gain of the worst-case system in the model set. With the parameter-integrated passivity, the quantitative analysis of a feedback system composed of two passive subsystems is provided as the parameter transition. Then, we find conditions on the matrix parameters to achieve the performance improvement such that the L_2-gain of the model set describing the feedback system is strictly less than that describing the subsystems. Subsequently, the performance improvement of the feedback system is extended to that of an iterative feedback system, which is a network system constructed by the feedback connection of multiple subsystems in a step-by-step manner. Then, we find conditions on the passivity parameters describing the baseline subsystem to achieve a gradual performance improvement with the subsystem connection.**

***Index Terms*—Passivity, Dissipativity, Model-set-based analysis, L_2-gain**

I. Introduction

THE control problems for large-scale systems have been extensively studied in the last several decades. A typical example is power system design. Power systems in recent years have been gradually incorporating a large number of renewable energy (RE) resources, and have therefore more complex and larger in scale. At any step of the construction, the demand and supply balance in the entire power system must be maintained. We need to develop the design and control strategy in a step-by-step manner for large-scale systems.

Let us consider the case where a baseline system is envolved by gradually connecting multiple subsystems in a step-by-step manner as illustrated in Fig. 1. Then, the multiple subsystems are implemented into a baseline system in a decentralized manner. In addition, the implementation proceeds as *a module connection*. For practical reasons, not only the implementation but also the design process should be done in a *modularized* manner [1] (this is called *distributed* design [2]); each subsystem is designed independently of the others except for their brief specification. Despite the difficulty of the modularized design, it is necessary to achieve stability and sufficient control performance of the entire control system.

K. Urata, T. Ishizaki, and J. Imura are with Graduate School of Engineering, Tokyo Institute of Technology, 2-12-1, Ookayama, Meguro, Tokyo, Japan. e-mail: {urata, ishizaki, imura}@cyb.sc.e.titech.ac.jp

M. Inoue is with Faculty of Science and Technology, Keio University, 3-14-1, Hiyoshi, Kohoku-ku, Yokohama, Japan. e-mail: minoue@appi.keio.ac.jp

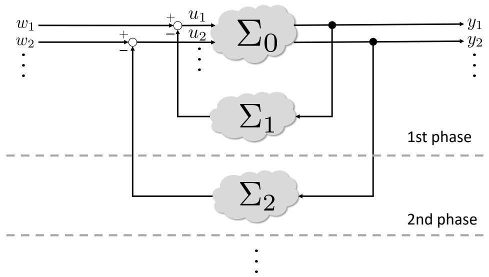

Fig. 1. Evolution of a baseline system Σ_0 by gradually connecting multiple subsystems Σ_i in a step-by-step manner.

Passivity is one of the key properties for realizing the modularized design of large-scale systems [1]. The passivity theorem [3] states that the feedback system composed of two passive systems is also passive. In Fig. 1, we consider the case where the baseline system and each subsystem have the passivity property. For the design of the subsystems stabilizing the entire control system, we do not require any models of the baseline system and subsystems, but require only their passivity instead. In fact, many papers [4]–[12] address passivity-based analysis and design, e.g., cooperative control [4]–[6], synchronization problem [7], stability analysis of power systems [9]–[11], and biological network analysis [12]. Passivity is utilized in these studies for *qualitative* analysis such as stability, consensus, and synchronization.

Other papers [13]–[16] address the *quantitative* analysis of feedback systems composed of two passive systems. The papers integrate scalar parameters into the passivity definition to quantify the passivity level, e.g., γ-passivity [13] and passivity indices [15], [16]. By employing the scalar parameters, the papers are able to derive more detailed quantitative analysis than the original passivity theorem [3]. For example, L_2-gain evaluation is performed in [14], [16].

In this paper, we address the problem of the *performance improvement* via the feedback connection of passive systems. Matrix parameters are integrated into the passivity, and they contribute to characterize passive systems more flexibly than the scalar parameters integrated in previous works [13]–[16]. The L_2-gain of the model set describing passive systems is defined as the worst-case system in the model set. We connect two passive systems described by the model set, to construct a passive feedback system. The feedback connection of the performance-integrated passive systems inherits the passivity in the disconnected case. The quantitative analysis of the feedback system is provided as the parameter transition. Then, we find conditions on the passivity parameters to achieve the performance improvement such that the L_2-gain of the model set describing the feedback system is strictly less than that describing the subsystems. It should be emphasized that only the passivity parameters characterizing each subsystem is utilized for the quantitative analysis of the feedback system, while the detailed model of the subsystem is not utilized. In this sense, the problem addressed in this paper is the *model-set-based* analysis of the feedback system.

Subsequently, the performance improvement of the feedback system is extended to that of an *iterative* feedback system, which is a network system constructed by the feedback connection of multiple subsystems in a step-by-step manner. The extension is motivated by its applicability to the modularized design of connecting subsystems in a step-by-step manner as illustrated in Fig. 1. By employing the matrix parameters, this paper is able to find a model set describing subsystems for a gradual improvement of the iterative feedback system, while the previous works are unable to do it. The paper [17] is a preliminary version of this paper. In [17], to achieve the performance improvement via the feedback connection, some passivity parameters for each subsystem is specialized to zero. This paper derives more general conditions on the passivity parameters to achieve the performance improvement.

The rest of the paper is organized as follows. In Section II-A, we show an example to motivate the performance improvement analysis through the frequency control of power systems with a large number of RE resources. In Section II-B, a general problem setting in this paper is introduced. The definition of performance-integrated passivity is provided in Section II-C. The performance analyses of general feedback and iterative feedback systems are presented in Section III. In this section, a solution to the problem formulated in Section II-B is provided. In other words, we find a model set of subsystems such that the performance improvement is achieved. Finally, the paper is concluded in Section IV.

Notation: $\mathbb{R}_+ := [0, \infty)$. The symbols $\mathbb{L}_2$ and $\mathbb{L}_{2e}$ denote the L_2-space and the extended L_2-space, respectively. For $\xi \in \mathbb{L}_2$, the symbol $\|\xi\|_{L_2}$ denotes the L_2-norm. For a causal and L_2-stable system Σ, the symbol $\|\Sigma\|_{L_2}$ denotes the L_2-gain. The symbol $\{M\}_{ij}$ denotes the (i, j) entry of a matrix M. For a real symmetric matrix X, the symbols $\underline{\lambda}(X)$ and $\bar{\lambda}(X)$ denote the minimum and maximum eigenvalues of X, respectively. The symbol $\underline{\mathbb{V}}(X)$ denotes the eigenspace corresponding to $\underline{\lambda}(X)$. The symbol $X^\dagger$ is said to be a generalized inverse of X if it satisfies $XX^\dagger X = X$. Furthermore, the matrix X is said to be irreducible if there does not exist any permutation matrix U such that $U^\top XU$ is block upper triangular. Two linear spaces $\mathcal{X}$ and $\mathcal{Y}$ are said to be disjoint if $\mathcal{X} \cap \mathcal{Y} = \{0\}$ holds. For a matrix $M \in \mathbb{R}^{m\times n}$ and a causal operator $\mathcal{F} : \mathbb{L}_{2e} \to \mathbb{L}_{2e}$, their direct product is defined as

$$M \otimes \mathcal{F} := \begin{bmatrix} \{M\}_{11}\mathcal{F} & \cdots & \{M\}_{1n}\mathcal{F} \\ \vdots & & \vdots \\ \{M\}_{m1}\mathcal{F} & \cdots & \{M\}_{mn}\mathcal{F} \end{bmatrix}.$$

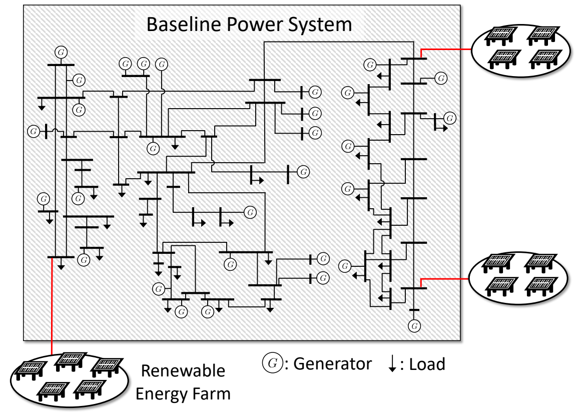

Fig. 2. A power network system is composed of some renewable energy farms, which comprise a large number of RE resources, and a baseline power system, which includes generators and loads.

II. Problem Setting: Model-set-based quantitative analysis

A. Motivating Example

We consider a power network system that is constructed by a baseline power system and some RE farms, as illustrated in Fig. 2. The baseline power system is represented by the IEEJ EAST 30-machine power model, which is the power system in the eastern half of Japan [18] with 30 generators, 31 loads, and 107 buses. The disturbance responses of the power system are illustrated in Fig. 3. From the figure, the disturbance effect is suppressed with the increase in the number of RE farms. The key to realizing the *performance improvement* is the *passivity* property, which is formally defined in Section II-C. It should be noted that the baseline power system has a *passivity* property; see Appendix A for details. In addition, the dynamics of the controlled RE farms has also *passivity* property. The improvement demonstrated in this example can be neither shown nor explained by directly applying the previous works [14]–[16] to this example. From this fact, we aim to find a special class of the passivity describing subsystems such that the performance improvement is achieved.

B. Model-set-based Performance Improvement Problem

We consider a feedback system $\Sigma_{\rm FB}(\Sigma_1, \Sigma_2)$ composed of two subsystems:

$$\Sigma_i : y_i = \mathcal{S}_i u_i,\ i \in \{1, 2\}, \tag{1}$$

where $\mathcal{S}_i : \mathbb{L}_{2e} \to \mathbb{L}_{2e}$ is a causal operator, and u_i and y_i denote the input and output of Σ_i, respectively. For example, suppose that Σ_i is a linear time-invariant (LTI) dynamical system. Then, $\mathcal{S}_i$ is given by the transfer function denoted by $\mathcal{S}_i(s)$ such that $y_i(s) = \mathcal{S}_i(s)u_i(s)$ holds, where $u_i(s)$ and $y_i(s)$ are the Laplace transforms of u_i and y_i, respectively. To construct the feedback system, Σ_1 and Σ_2 are internally connected via negative feedback. Let $w \in \mathbb{R}^m$ and $z \in \mathbb{R}^m$ be the external input and control output of $\Sigma_{\rm FB}(\Sigma_1, \Sigma_2)$, respec-

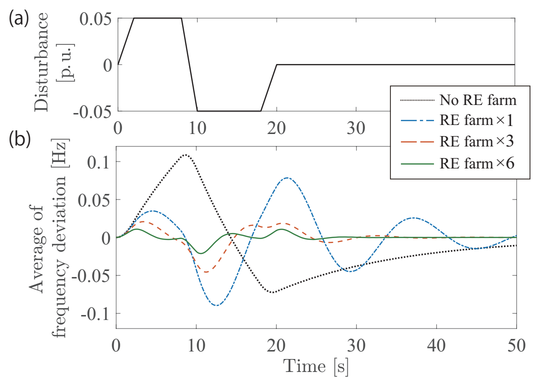

Fig. 3. Disturbance responses of the power system constructed by connecting RE farms to the baseline power system in a step-by-step manner. (a) The disturbance is injected into all RE farms of the power system. (b) The resulting output, which is the average of all frequency deviations, is illustrated. The output is increasingly suppressed with the increase in the number of the connected RE farms.

tively. Then, the negative feedback connection is described as

$$u_1 = w - y_2, \tag{2}$$

$$u_2 = y_1 = z. \tag{3}$$

In this paper, we assume that $\Sigma_{\rm FB}(\Sigma_1, \Sigma_2)$ is well-posed, i.e., y_1 and y_2 of $\Sigma_{\rm FB}(\Sigma_1, \Sigma_2)$ uniquely exist and belong to $\mathbb{L}_{2e}$ for all $w \in \mathbb{L}_{2e}$.

To formulate the problem of the performance improvement, which is the main contribution of this paper, we define some model sets and its performance measure. Let $\mathcal{P}_i, i \in \{1, 2\}$ be model set describing each subsystem $\Sigma_i, i \in \{1, 2\}$, namely, the set of Σ_i satisfying some specific property. Then, the model set describing the feedback system $\Sigma_{\rm FB}$ is defined as

$$\mathcal{P}_{\rm FB}(\mathcal{P}_1, \mathcal{P}_2) := \{\Sigma_{\rm FB}(\Sigma_1, \Sigma_2) | \Sigma_i \in \mathcal{P}_i, i \in \{1, 2\}\}.$$

For example, let us consider Σ_1 and Σ_2 are single-input-single-output (SISO) systems. As illustrated in Fig. 4a, each model set $\mathcal{P}_i, i \in \{1, 2\}$ describing Σ_i is represented by the set of all Nyquist plots of $\Sigma_i \in \mathcal{P}_i$. Then, the set of Nyquist plots of $\Sigma_{\rm FB}$, which represents the model set $\mathcal{P}_{\rm FB}(\mathcal{P}_1, \mathcal{P}_2)$, is illustrated in Fig. 4b. This paper addresses the quantitative analysis, such as the L_2-gain evaluation, of the *model set* describing $\Sigma_{\rm FB}$. To this end, the L_2-gain of the *model set* $\mathcal{P}_i$ is defined as follows.

Definition 1: Suppose that all Σ_i belonging to $\mathcal{P}_i$ are L_2-stable. Then,

$$\gamma(\mathcal{P}_i) := \sup_{\Sigma_i \in \mathcal{P}_i} \|\Sigma_i\|_{L_2}$$

is said to be the L_2-gain of $\mathcal{P}_i$.

We see that the L_2-gain of the *model set* $\mathcal{P}_i$ is defined as the worst-case system in $\mathcal{P}_i$. In this paper, we address the following problem of performance improvement.

Problem 1: Find $\mathcal{P}_i, i \in \{1, 2\}$ such that

$$\gamma(\mathcal{P}_{\rm FB}(\mathcal{P}_1, \mathcal{P}_2)) < \gamma(\mathcal{P}_1) \tag{4}$$

holds.

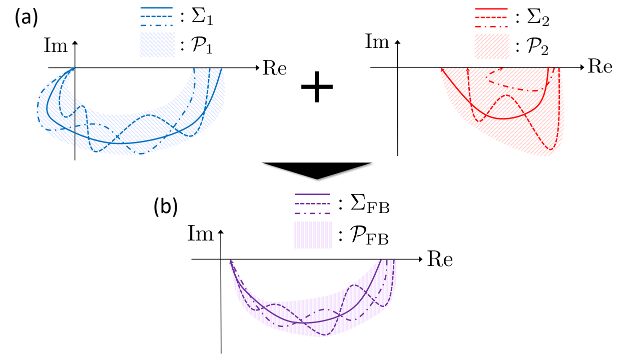

Fig. 4. Example of model set for SISO systems: (a) Nyquist plots of $\Sigma_i \in \mathcal{P}_i, i \in \{1, 2\}$. (b) Nyquist plots of $\Sigma_{\rm FB} \in \mathcal{P}_{\rm FB}$, which are generated from $\Sigma_i \in \mathcal{P}_i, i \in \{1, 2\}$ in subfigure a.

Problem 1 can be called *model-set-based analysis* of the feedback system $\Sigma_{\rm FB}$. Each subsystem $\Sigma_i, i \in \{1, 2\}$ and $\Sigma_{\rm FB}$ are described by model sets $\mathcal{P}_i$ and $\mathcal{P}_{\rm FB}$, respectively, rather than precise models such as a state-space model and a transfer function. In the model-set-based analysis, only the model set describing Σ_i is utilized for the analysis of $\Sigma_{\rm FB}$, while the precise model of Σ_i is not utilized at all. In particular, the aim of this paper is to find $\mathcal{P}_i$ to achieve the performance improvement (4), i.e., the L_2-gain of $\mathcal{P}_{\rm FB}(\mathcal{P}_1, \mathcal{P}_2)$ decreases as compared to that of $\mathcal{P}_1$.

C. Definition of Dissipativity and Passivity

One of the key tools to describe input-output systems is dissipativity, in particular, $(Q, S, R)-$dissipativity [19], [20], which is defined as follows.

Definition 2: Let $Q = Q^\top \in \mathbb{R}^{m\times m}$, $S \in \mathbb{R}^{m\times m}$, and $R = R^\top \in \mathbb{R}^{m\times m}$. Then, the system Σ_i is said to be (Q, S, R)-dissipative if there exists $\rho_i \in \mathbb{R}$ such that for any $u_i \in \mathbb{L}_{2e}$ and its corresponding output y_i,

$$\int_0^T \left\{ u_i^\top(\tau) S y_i(\tau) - \frac{1}{2} y_i^\top(\tau) Q y_i(\tau) - \frac{1}{2} u_i^\top(\tau) R u_i(\tau) \right\} d\tau \geq \rho_i \tag{5}$$

holds for all $T \in \mathbb{R}_+$.

The (Q, S, R)-dissipativity characterizes the set of dynamical systems by three parameters Q, S, and R. The parameters have been utilized for the *qualitative* stability analysis of large-scale systems [3]–[6], [21]. The aim of this paper is to find a special class of (Q, S, R)-dissipative subsystems to achieve the performance improvement. Even if the L_2-gain of the (Q, S, R)-dissipativity is characterized by the parameters Q, S, and R, we cannot show the performance improvement (4) for the general parameters. In this paper, we restrict the parameters in the (Q, S, R)-dissipativity to solve Problem 1 and focus on the passivity [20], [22].

Definition 3: The system Σ_i is said to be (Q, R)-passive if it is (Q, S, R)-dissipative with respect to $Q \geq 0, S = I_m$, and $R \geq 0$. In addition, the set of all (Q, R)-passive systems is denoted by $\mathcal{P}(Q, R)$.

In the definition, we remark that $Q^{-1} \geq R$ necessarily holds if the model-set $\mathcal{P}(Q,R)$ is not empty. The matrix parameters Q and R characterize the model set $\mathcal{P}(Q,R)$ and express the passivity level of a dynamical system. In the following discussion, we confine our attention of the model set $\mathcal{P}_i$ in Problem 1 to $\mathcal{P}(Q_i,R_i)$, i.e., $\mathcal{P}_i = \mathcal{P}(Q_i,R_i), i \in \{1,2\}$ holds. This means that the (Q,R)-passivity property is imposed on Σ_i.

III. Performance Analysis and Improvement of Passive Systems

In this section, a solution to Problem 1 is provided. We first provide the performance evaluation of L_2-gain of $\mathcal{P}(Q_1,R_1)$ and $\mathcal{P}_{\rm FB}(\mathcal{P}_1,\mathcal{P}_2)$, respectively. On the basis of the performance evaluation, a model set describing subsystems $\Sigma_i, i \in \{1,2\}$ for the performance improvement of the feedback system is derived. In other words, we find Q_i and $R_i, i \in \{1,2\}$ such that (4) holds. Subsequently, the performance improvement of the feedback system is extended to that of an *iterative* feedback system, which is defined in Subsection III-B.

A. Performance Analysis and Improvement of Feedback Systems

We introduce a performance index to evaluate the L_2-gain of the model set $\mathcal{P}(Q,R)$ as

$$\varepsilon(Q,R) := \frac{1+\sqrt{\underline{\lambda}(Q)\bar{\lambda}(Q^{-1}-R)}}{\underline{\lambda}(Q)} \tag{6}$$

Then, the performance evaluation of the L_2-gains of $\mathcal{P}(Q_1,R_1)$ and $\mathcal{P}_{\rm FB}$ is provided in the following lemmas.

Lemma 1: Suppose that $Q_1 > 0$ holds. Then, the L_2-gain of $\mathcal{P}(Q_1,R_1)$ satisfies

$$\gamma(\mathcal{P}_1) = \varepsilon(Q_1,R_1), \tag{7}$$

where $\varepsilon(Q_1,R_1)$ is given by (6)

Lemma 2: Suppose that $\mathcal{P}_i = \mathcal{P}(Q_i,R_i), i \in \{1,2\}$ holds. Then, letting $Q_{\rm FB} \geq 0$ and $R_{\rm FB} \geq 0$ be given by

$$Q_{\rm FB} = Q_1 + R_2, \tag{8}$$

$$R_{\rm FB} = R_1(Q_2+R_1)^{\dagger}Q_2, \tag{9}$$

it holds that $\mathcal{P}_{\rm FB}(\mathcal{P}_1,\mathcal{P}_2) \subseteq \mathcal{P}(Q_{\rm FB},R_{\rm FB})$. In addition, if $Q_{\rm FB} > 0$ holds, the L_2-gain of $\mathcal{P}_{\rm FB}(\mathcal{P}_1,\mathcal{P}_2)$ satisfies

$$\gamma(\mathcal{P}_{\rm FB}) \leq \varepsilon(Q_{\rm FB},R_{\rm FB}), \tag{10}$$

where $\varepsilon(\cdot,\cdot)$ is given by (6).

The proofs of Lemmas 1 and 2 are given in Appendix B and C, respectively. Lemma 1 provides the performance evaluation of the L_2-gain of $\mathcal{P}(Q_1,R_1)$. In particular, $\gamma(\mathcal{P}(Q_1,R_1))$ is *tightly* evaluated by (6), which is determined only by the passivity parameters Q_1 and R_1. This implies that the evaluation does not require the detailed models of Σ_1. In Lemma 2, an outer-approximation of $\mathcal{P}_{\rm FB}(\mathcal{P}_1,\mathcal{P}_2)$ is given by $\mathcal{P}(Q_{\rm FB},R_{\rm FB})$, which is explicitly characterized by the parameter transition in (8) and (9). Suppose that $\Sigma_{\rm FB}$ is described by a SISO system. Then, $\mathcal{P}(Q_{\rm FB},R_{\rm FB})$ is represented by the disk

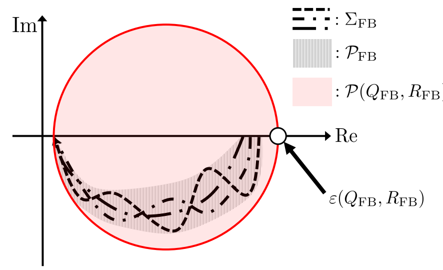

Fig. 5. Outer-approximation of $\mathcal{P}_{\rm FB}$ by $\mathcal{P}(Q_{\rm FB},R_{\rm FB})$, which is represented by the disk region in the closed right-half plane of the complex plane. The performance index $\varepsilon(Q_{\rm FB},R_{\rm FB})$ is graphically obtained and is depicted by the white circle.

region that includes the model set $\mathcal{P}_{\rm FB}(\mathcal{P}_1,\mathcal{P}_2)$, as illustrated in Fig. 5. With the transition, we evaluate the L_2-gain of $\mathcal{P}_{\rm FB}$ in the sense of the upper bound $\varepsilon(Q_{\rm FB},R_{\rm FB})$. On the basis of Lemmas 1 and 2, a solution to Problem 1 is provided in the following theorem.

Theorem 1: Suppose that $Q_1 > 0$ holds in addition to the condition of Lemma 2. Then, the following two statements hold:

i) Suppose that $Q_2 = q_2 I_m > 0$, $R_1 = r_1 I_m > 0$, and $q_2\underline{\lambda}(R_2) \geq \underline{\lambda}(Q_1)r_1$ hold. Then, (4) holds.
ii) Suppose that either $Q_2 = 0$ or $R_1 = 0$ holds. Then, $\underline{\mathbb{V}}(Q_1)$ and $\operatorname{Ker} R_2$ are disjoint if and only if (4) holds.

Proof: i) Suppose that $R_1 = r_1 I_m > 0$ and $Q_2 = q_2 I_m \geq 0$ hold. From Lemma 2, $\mathcal{P}_{\rm FB} \subseteq \mathcal{P}(Q_{\rm FB},R_{\rm FB})$ holds, where $Q_{\rm FB}$ and $R_{\rm FB}$ are given in (8) and (9). In addition, from Lemma 1, $\varepsilon(Q_1,R_1)$ and $\varepsilon(Q_{\rm FB},R_{\rm FB})$ are given by

$$\varepsilon(Q_1,R_1) = \frac{1+\sqrt{1-\underline{\lambda}(Q_1)r_1}}{\underline{\lambda}(Q_1)},$$

$$\varepsilon(Q_{\rm FB},R_{\rm FB}) = \frac{1+\sqrt{1-\underline{\lambda}(Q_1+R_2)\dfrac{q_2 r_1}{q_2+r_1}}}{\underline{\lambda}(Q_1+R_2)}.$$

Note that $\underline{\lambda}(Q_1+R_2) \geq \underline{\lambda}(Q_1) + \underline{\lambda}(R_2)$ and suppose that $q_2\underline{\lambda}(R_2) \geq \underline{\lambda}(Q_1)r_1$ holds. Then, we show that

$$\begin{aligned}&\underline{\lambda}(Q_1+R_2)\frac{q_2 r_1}{q_2+r_1} - \underline{\lambda}(Q_1)r_1 \\ &= \frac{q_2 r_1}{q_2+r_1}(\underline{\lambda}(Q_1+R_2)-\underline{\lambda}(Q_1)) - \frac{r_1^2}{q_2+r_1}\underline{\lambda}(Q_1) \\ &\geq \frac{r_1}{q_2+r_1}(q_2\underline{\lambda}(R_2)-\underline{\lambda}(Q_1)r_1) \geq 0\end{aligned} \tag{11}$$

holds. Noting that $q_2/(q_2+r_1) < 1$ holds in (11), we have

$$\underline{\lambda}(Q_1+R_2) > \underline{\lambda}(Q_1). \tag{12}$$

From (11) and (12), we show that

$$\varepsilon(Q_{\rm FB},R_{\rm FB}) < \varepsilon(Q_1,R_1) \tag{13}$$

holds. It follows that (4) holds.

ii) Suppose that either $Q_2 = 0$ or $R_1 = 0$ holds. Then, $\varepsilon(Q_1,R_1)$ and $\varepsilon(Q_{\rm FB},R_{\rm FB})$ are given by

$$\varepsilon(Q_1,R_1) = \frac{2}{\underline{\lambda}(Q_1)}, \quad \varepsilon(Q_{\rm FB},R_{\rm FB}) = \frac{2}{\underline{\lambda}(Q_1+R_2)}.$$

From Lemma 3 given in Appendix D, $\underline{\mathbb{V}}(Q_1)$ and $\mathrm{Ker}\, R_2$ are disjoint if and only if (12) holds. This is equivalent to (13). It follows that (4). This completes the proof of Theorem 1. ∎

Theorem 1, which gives a solution to Problem 1, provides a model set describing subsystems such that the performance improvement is achieved. This is one of the main contribution of this paper.

Remark 1: Suppose that Σ_i in (1) is L_2-stable. In addition, assume that the initial state of Σ_i is zero. Then, for any $u_i \in \mathbb{L}_2$ and its corresponding output y_i, the inequality (5) holds even if $T \to \infty$ and $\rho_i = 0$. Applying Parseval's theorem to the inequality (5), as $T \to \infty$, we obtain the following expression. The system Σ_i is said to be the *frequency-dependent* (Q, R)-passive if, for any $u_i \in \mathbb{L}_2$ and its corresponding output y_i, it holds that

$$\int_{-\infty}^{\infty} \left\{ \hat{u}_i^\top(j\omega)\hat{y}_i(j\omega) - \frac{1}{2}\hat{y}_i^\top(j\omega)Q(\omega)\hat{y}_i(j\omega) - \frac{1}{2}\hat{u}_i^\top(j\omega)R(\omega)\hat{u}_i(j\omega) \right\} d\omega \geq 0, \quad (14)$$

where $\hat{u}_i$ and $\hat{y}_i$ denote the Fourier transforms of u_i and y_i, respectively, and $Q(\omega)$ and $R(\omega)$ are bounded and positive semidefinite matrix-valued frequency functions. When $Q(\omega)$ and $R(\omega)$ are independent of ω, i.e., $Q(\omega) = Q$ and $R(\omega) = R$, (14) is reduced to (5). Let $\mathcal{P}_\omega(Q(\omega), R(\omega))$ be the set of dynamical systems that satisfy (14) for some $Q(\omega)$ and $R(\omega)$. We see that $\mathcal{P}_\omega(Q(\omega), R(\omega))$ is a more general set than $\mathcal{P}(Q, R)$. By introducing the frequency-dependent passivity, we address the problem of the performance improvement in the sense of the *frequency-dependent gain* of the model set. Assuming that Σ_i is LTI and recalling its transfer function representation $\mathcal{S}_i(s)$, we define the gain of $\mathcal{P}_i = \mathcal{P}_\omega(Q_i(\omega), R_i(\omega))$ as

$$\tilde{\gamma}(\mathcal{P}_i, \omega) := \sup_{\mathcal{S}_i \in \mathcal{P}_i} \bar{\sigma}(\mathcal{S}_i(j\omega)).$$

Suppose that $\mathcal{P}_1 = \mathcal{P}_\omega(Q_1(\omega), R_1(\omega))$ and $Q_1(\omega) > 0$ for all $\omega \in \mathbb{R}$. Then, we show that the gains of $\mathcal{P}_1$ and $\mathcal{P}_{\mathrm{FB}}$ satisfies

$$\tilde{\gamma}(\mathcal{P}_1, \omega) = \varepsilon(Q_1(\omega), R_1(\omega)), \quad \forall\omega \in \mathbb{R},$$
$$\tilde{\gamma}(\mathcal{P}_{\mathrm{FB}}, \omega) \leq \varepsilon(Q_{\mathrm{FB}}(\omega), R_{\mathrm{FB}}(\omega)), \quad \forall\omega \in \mathbb{R},$$

where $Q_{\mathrm{FB}}(\omega)$ and $R_{\mathrm{FB}}(\omega)$ are given by (8) and (9), and $\varepsilon(\cdot,\cdot)$ is given by (6). In a similar manner to the analysis in Theorem 1, we can find $Q_i(\omega)$ and $R_i(\omega)$ to achieve the performance improvement for *all frequency points*, i.e., it follows that

$$\tilde{\gamma}(\mathcal{P}_{\mathrm{FB}}, \omega) < \tilde{\gamma}(\mathcal{P}_1, \omega)$$

for all $\omega \in \mathbb{R}$. Furthermore, letting $Q_i(\omega)$ and $R_i(\omega)$ be

$$Q_i(\omega) = \begin{cases} Q_i, & \omega \in \Omega \\ 0, & \text{otherwise}, \end{cases} \qquad R_i(\omega) = \begin{cases} R_i, & \omega \in \Omega \\ 0, & \text{otherwise}, \end{cases}$$

we restrict the infinite integration interval in (14) to a finite frequency range Ω. Then, we can find Q_i and R_i to achieve the performance improvement in the range Ω of the feedback system that is composed of even non-passive subsystems.

B. Performance Analysis and Improvement of Iterative Feedback Systems

We extend the model-set-based performance analysis in Theorem 1 to that of the iterative feedback system, which is a network system constructed by the feedback connection of multiple subsystems in a step-by-step manner. In the iterative feedback system, we assume that each connected subsystem is implemented in a decentralized manner as illustrated in Fig. 1. The aim of this subsection is to find a model set describing connected subsystems to achieve a *gradual* improvement of the performance of the iterative feedback system with the implementation progress. For simplicity of analysis, we consider the case where one subsystem is connected to a baseline system Σ_0. Then, we provide a port selection of the subsystem connection for the performance improvement such that the L_2-gain of the model set describing the overall control system is strictly less than that describing the connected subsystem.

Let us consider a baseline system Σ_0 described by

$$\Sigma_0 : y = \mathcal{S}_0 u, \quad (15)$$

where $\mathcal{S}_0 : \mathbb{L}_{2e} \to \mathbb{L}_{2e}$ is a causal operator, and $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$ denote the input and output of Σ_0, respectively. For Σ_0 in (15), let y be partitioned as $y =: [\, y_1^\top \ y_2^\top \ \cdots \ y_m^\top \,]^\top$, where the subscript ℓ of $y_\ell \in \mathbb{R}$ represents the port number. The subsystem Σ_ℓ is connected to the ℓ-th port of Σ_0. Then, Σ_ℓ is described as

$$\Sigma_\ell : v_\ell = \mathcal{S}_\ell y_\ell, \quad (16)$$

where $\mathcal{S}_\ell : \mathbb{L}_{2e} \to \mathbb{L}_{2e}$ is a causal operator and $v_\ell \in \mathbb{R}$ denotes the output of Σ_ℓ. We define an operator $\mathcal{F}_\ell$ as

$$\mathcal{F}_\ell = E_\ell \otimes \mathcal{S}_\ell, \quad (17)$$

where $E_\ell = e_\ell e_\ell^\top \in \mathbb{R}^{m\times m}$. The symbol e_ℓ denotes the unit vector in which the ℓ-th entry is one, while the others are zero. Using the input-output operator of (17), we rewrite each subsystem Σ_ℓ to an another subsystem Π_ℓ, which is described as

$$\Pi_\ell : v = \mathcal{F}_\ell y, \quad (18)$$

where $v = [\, v_1^\top \ v_2^\top \ \cdots \ v_m^\top \,]^\top$. From (17) and (18), we see that Σ_ℓ has a *decentralized structure*, which is the result of expressing the implementation of subsystem Σ_ℓ in a decentralized manner as that in a centralized manner. The feedback connection of Π_ℓ to Σ_0 is described by

$$u = w - v, \quad (19)$$
$$z = y. \quad (20)$$

We construct the entire control system with the input w and output z by connecting Π_ℓ to Σ_0 via the negative feedback of (19) and (20). The entire system is denoted by $\Sigma_{\mathrm{ent}}(\Sigma_0, \Sigma_\ell)$. Let $\mathcal{P}_0$ and $\mathcal{P}_\ell$ be the model sets describing Σ_0 and Σ_ℓ, respectively. Then, the model set describing $\Sigma_{\mathrm{ent}}(\Sigma_0, \Sigma_\ell)$ is described as

$$\mathcal{P}_{\mathrm{ent},\ell} := \mathcal{P}_{\mathrm{FB}}(\mathcal{P}_0, \mathcal{P}_\ell).$$

We consider the case where Σ_0 is passive and strictly proper. Then, $\mathcal{P}_0 = \mathcal{P}(Q, 0)$ holds for some $Q \geq 0$. The following theorem shows the performance improvement of $\Sigma_{\rm ent}(\Sigma, \Sigma_\ell)$.

Theorem 2: Suppose that $\mathcal{P}_0 = \mathcal{P}(Q, 0)$ and $\mathcal{P}_\ell = \mathcal{P}(q_\ell E_\ell, r_\ell E_\ell)$ hold for some $q_\ell \geq 0$ and $r_\ell \geq 0$. Then, for $Q_{{\rm ent},\ell} \geq 0$ given by

$$Q_{{\rm ent},\ell} = Q + r_\ell E_\ell, \tag{21}$$

it holds that $\mathcal{P}_{{\rm ent},\ell} \subseteq \mathcal{P}(Q_{{\rm ent},\ell}, 0)$. Furthermore, suppose that $Q > 0$ and $r_\ell > 0$ hold. Then,

$$\gamma(\mathcal{P}_{{\rm ent},\ell}) < \gamma(\mathcal{P}_0) \tag{22}$$

holds if and only if the ℓ-th entry of any $v \in \underline{\mathbb{V}}(Q)$ is nonzero.

Proof: From Lemma 2, we see that $\mathcal{P}_{{\rm ent},\ell} \subseteq \mathcal{P}(Q_{{\rm ent},\ell}, 0)$ holds. Next, we show that (22) holds if and only if the ℓ-th entry of any $v \in \underline{\mathbb{V}}(Q)$ is nonzero.

(Necessary condition for (22)) Suppose that $Q > 0$, $r_\ell > 0$, and (22) hold, and equivalently

$$\underline{\lambda}(Q_{{\rm ent},\ell}) > \underline{\lambda}(Q) \tag{23}$$

holds. From Lemma 3, it follows that $\underline{\mathbb{V}}(Q)$ and ${\rm Ker}\, E_\ell$ are disjoint and equivalently for any $v \in \underline{\mathbb{V}}(Q)$, $v \notin {\rm Ker}\, E_\ell \setminus \{0\}$ holds. Because ${\rm Ker}\, E_\ell = {\rm span}\{e_1, \ldots, e_{\ell-1}, e_{\ell+1}, \ldots, e_m\}$ holds, we show that the ℓ-th entry of any $v \in \underline{\mathbb{V}}(Q)$ is nonzero.

(Sufficient condition for (22)) Suppose that the ℓ-th entry of any $v \in \underline{\mathbb{V}}(Q)$ is nonzero. Then, $v \notin {\rm Ker}\, E_\ell \setminus \{0\}$ holds. This implies that $\underline{\mathbb{V}}(Q)$ and ${\rm Ker}\, E_\ell$ are disjoint. From Lemma 3, we show that $\underline{\lambda}(Q_{{\rm ent},\ell}) > \underline{\lambda}(Q)$. It follows that (22) holds. ■

As shown in Theorem 2, $\mathcal{P}_{{\rm ent},\ell}$ inherits the passivity property from $\mathcal{P}_0$ independently of the port selection of the subsystem connection. In addition, an outer-approximation of $\mathcal{P}_{{\rm ent},\ell}$ is given by $\mathcal{P}(Q_{{\rm ent},\ell}, 0)$, which is explicitly characterized by the parameter transition in (21). Furthermore, Theorem 2 provides a condition of port selection for the performance improvement such that the L_2-gain of $\mathcal{P}_{{\rm ent},\ell}$ decreases compared to that of $\mathcal{P}(Q, 0)$. In Theorems 1 and 2, we provide a condition of the passivity parameters for the performance improvement. Theorem 1 addresses the feedback system composed of general subsystems and provides a general condition for the improvement. Theorem 2 is a specialization of Theorem 1 by addressing the decentralized structure in the connected subsystems.

Theorem 2 is applicable to the performance improvement of the iterative feedback system. Note that the performance improvement by Theorem 2 is shown under the requirement for the connection port of the subsystem Σ_ℓ (equivalently subsystem Π_ℓ with the decentralized structure). This implies that the condition for the performance improvement is checked at every connection step. By restricting the class of Q, we show the performance improvement under a requirement only on Σ independently of the port selection.

Proposition 1: Suppose that $\mathcal{P}_0 = \mathcal{P}(Q, 0)$ and $\mathcal{P}_\ell = \mathcal{P}(q_\ell E_\ell, r_\ell E_\ell)$ hold for $Q \in \mathbb{M}$, $q_\ell \geq 0$, and $r_\ell > 0$, where $\mathbb{M}$ denotes the set of M-matrices [23]. Then, for any ℓ, $Q_{{\rm ent},\ell} \in \mathbb{M}$ and $\mathcal{P}_{{\rm ent},\ell} \subseteq \mathcal{P}(Q_{{\rm ent},\ell}, 0)$ hold. In addition, for any ℓ, (22) holds if and only if Q is irreducible.

The proof of Proposition 1 is given in Appendix E.

Proposition 1 provides a model set describing Σ_0 for the *gradual* performance improvement. From Proposition 1, the passivity parameter $Q_{{\rm ent},\ell}$ is the M-matrix independent of the port selection of the subsystem connection. In addition, the *irreducibility* of $Q_{{\rm ent},\ell}$ achieves the performance improvement for *any* port selection. From these facts, as long as the passivity parameter of Σ_0 is the irreducible M-matrix, the performance improvement is achieved whenever any ISP subsystem is connected to *any port* of the baseline system at *any step*. In this sense, Proposition 1 provides a class of Q for a *gradual* improvement of the L_2-gain of $\mathcal{P}(Q_{{\rm ent},\ell}, 0)$.

As stated in Remark 1, the analysis in this section can also be extended to a more general analysis where the passivity parameters are dependent on the frequency.

Remark 2: Proposition 1 requires a severe condition to be imposed on Σ_0 to show the improvement mathematically. The condition that Q is an M-matrix is technical. In most cases, $v \in \underline{\mathbb{V}}(Q)$ can have no zero entries even though Q is not an M-matrix. Actually, the *irreducibility* of Q plays an important role in the gradual improvement of the L_2-gain of $\mathcal{P}(Q_{{\rm ent},\ell}, 0)$.

IV. Conclusion

This paper addressed the model-set-based quantitative analysis of the feedback system. In particular, we found the passivity parameters in the model sets that describes subsystems such that the performance improvement is achieved; the L_2-gain of the model set that describes the feedback system decreases as compared to that describes the disconnected subsystem. A solution to the problem was provided by Theorem 1. Furthermore, the model-set-based performance analysis and improvement of the feedback system were extended to those of an iterative feedback system with a decentralized structure. Then, a condition on the passivity parameter for gradual improvement of the L_2-gain of the model set was derived.

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Appendix

A. Power System Model and Simulation Setup

In the IEEJ EAST 30-machine model, each generator model consists of a simplified synchronous machine described as a second-order dynamics and a first-order turbine-governor dynamics [24]. We suppose that there are no isolated buses in the baseline power system, i.e., the topology of the network in the baseline power system is described by the strongly connected graph [25]. The interconnection between some connected buses can be represented as the power flow equation [24] with an assumption of a lossless network. We consider the linearized baseline power system around an equilibrium point. The input and output of the linearized power system are the output of RE farms and the weighted sum of frequency deviation in all synchronous generators, respectively. Then, the weighting factor is determined such that the baseline system is passive, which can be evaluated by, e.g. the KYP lemma [26]. The output of RE farms is generated by the PI controller, where the proportional and integral gains are selected from values in the range of $[1\ 4]$. In addition, the disturbance is additively injected into the output of the RE farms. The entire power system is represented by the feedback system composed of the baseline power system and the RE farms.

B. Proof of Lemma 1

Let $S_1 = 0$ and $\tilde{y}_1 := y_1 - Q_1^{-1}u_1$. Then, it follows from (5) that for any $u_1 \in \mathbb{L}_{2e}$,

$$\int_0^T \tilde{y}_1^\top(\tau)Q_1\tilde{y}_1(\tau)d\tau \le \int_0^T u_1^\top(\tau)(Q_1^{-1}-R_1)u_1(\tau)d\tau - \rho_1.$$

holds. Because $u_1^\top(Q_1^{-1}-R_1)u_1 \le \bar{\lambda}(Q_1^{-1}-R)u_1^\top u_1$ and $\underline{\lambda}(Q_1)\tilde{y}_1^\top\tilde{y}_1 \le \tilde{y}_1^\top Q_1\tilde{y}_1$ hold, we have that

$$\begin{aligned}&\underline{\lambda}(Q_1)\int_0^T \tilde{y}_1{}^\top(\tau)\tilde{y}_1(\tau)d\tau\\ &\quad\le \bar{\lambda}(Q_1^{-1}-R_1)\int_0^T u_1^\top(\tau)u_1(\tau)d\tau - \rho_1\end{aligned}$$

holds for all $T \in \mathbb{R}_+$. It follows that

$$\|\tilde{y}_1\|_{L_2,T} \le \frac{\sqrt{\underline{\lambda}(Q_1)\bar{\lambda}(Q_1^{-1}-R_1)}}{\underline{\lambda}(Q_1)}\|u_1\|_{L_2,T} - \frac{1}{\underline{\lambda}(Q_1)}\rho_1$$

for all $u_1 \in \mathbb{L}_{2e}$ and $T \in [0,\infty)$, where $\|\cdot\|_{L_2,T}$ denotes the finite time L_2-norm: for $\xi \in \mathbb{L}_2$ and $T \in \mathbb{R}_+$,

$$\|\xi\|_{L_2,T} := \left(\int_0^T \|\xi(\tau)\|^2 d\tau\right)^{\frac{1}{2}}.$$

Recalling that $y_1 = \tilde{y}_1 + Q_1^{-1}u_1$, we have

$$\begin{aligned}\|y_1\|_{L_2,T} &\le \|\tilde{y}_1\|_{L_2,T} + \frac{1}{\underline{\lambda}(Q_1)}\|u_1\|_{L_2,T}\\ &\le \varepsilon(Q_1,R_1)\|u_1\|_{L_2,T} - \frac{1}{\underline{\lambda}(Q_1)}\rho_1.\end{aligned}$$

Because $\|u_1\|_{L_2,T} \le \|u_1\|_{L_2}$ holds for any $u_1 \in \mathbb{L}_2$, it follows that

$$\|y_1\|_{L_2,T} \le \varepsilon(Q_1,R_1)\|u_1\|_{L_2} - \frac{1}{\underline{\lambda}(Q_1)}\rho_1. \tag{24}$$

We see that (24) holds as $T \to \infty$. Then, $\Sigma_1 \in \mathcal{P}(Q_1,R_1)$ with $Q_1 > 0$ is L_2-stable, and (24) is reduced to

$$\|\Sigma_1\|_{L_2} \le \varepsilon(Q_1,R_1) \tag{25}$$

with $\varepsilon(Q_1,R_1)$ of (6).

Finally, to show (7), we aim to find an example of the "worst" Σ_1 in $\mathcal{P}(Q_1,R_1)$ such that (25) holds with the equality. To this end, let us consider Σ_1 described by the transfer function representation:

$$\begin{aligned}y_1(s) = \Bigg(&\frac{2\sqrt{\underline{\lambda}(Q_1)\bar{\lambda}(Q_1^{-1}-R_1)}}{\underline{\lambda}(Q_1)}\frac{1}{Ts+1}\\ &+ \frac{1-\sqrt{\underline{\lambda}(Q_1)\bar{\lambda}(Q_1^{-1}-R_1)}}{\underline{\lambda}(Q_1)}\Bigg)u_1(s),\end{aligned}$$

where $Q_1 > 0$ and $R_1 \ge 0$ with $Q_1^{-1} \ge R_1$. Then, we see that $\Sigma_1 \in \mathcal{P}(Q_1,R_1)$ holds. In addition, the performance criterion of $\mathcal{P}(Q_1,R_1)$ is given by (6), which is equivalent to the actual L_2-gain $\|\Sigma_1\|_{L_2}$. It follows that (7) holds. ■

C. Proof of Lemma 2

From the definition of passivity, we see that there exists $\rho_{\rm FB} \in \mathbb{R}$ such that the inequality

$$\begin{aligned}&\sum_{i=1}^2\int_0^T (2u_i^\top(\tau)y_i(\tau) - y_i^\top(\tau)Q_iy_i(\tau) - u_i^\top(\tau)R_iu_i(\tau))d\tau\\ &= \int_0^T\sum_{i=1}^2 \begin{bmatrix}u_i(\tau)\\ y_i(\tau)\end{bmatrix}^\top \begin{bmatrix}-R_i & I_m\\ I_m & -Q_i\end{bmatrix}\begin{bmatrix}u_i(\tau)\\ y_i(\tau)\end{bmatrix}d\tau \ge \rho_{\rm FB}\end{aligned}$$

holds for all $T \in \mathbb{R}_+$. Utilizing (2) and (3), we have

$$\begin{aligned}&\sum_{i=1}^2 \begin{bmatrix}u_i(\tau)\\ y_i(\tau)\end{bmatrix}^\top \begin{bmatrix}-R_i & I_m\\ I_m & -Q_i\end{bmatrix}\begin{bmatrix}u_i(\tau)\\ y_i(\tau)\end{bmatrix}\\ &= \begin{bmatrix}w(\tau)-y_2(\tau)\\ y_1(\tau)\end{bmatrix}^\top \begin{bmatrix}R_1 & I_m\\ I_m & -Q_1\end{bmatrix}\begin{bmatrix}w(\tau)-y_2(\tau)\\ y_1(\tau)\end{bmatrix}\\ &\quad+ \begin{bmatrix}y_1(\tau)\\ y_2(\tau)\end{bmatrix}^\top \begin{bmatrix}-R_2 & I_m\\ I_m & -Q_2\end{bmatrix}\begin{bmatrix}y_1(\tau)\\ y_2(\tau)\end{bmatrix}\\ &= \begin{bmatrix}w(\tau)\\ z(\tau)\\ y_2(\tau)\end{bmatrix}^\top \begin{bmatrix}-R_1 & I_m & R_1\\ I_m & -Q_1-R_2 & 0\\ R_1 & 0 & -Q_2-R_1\end{bmatrix}\begin{bmatrix}w(\tau)\\ z(\tau)\\ y_2(\tau)\end{bmatrix}.\end{aligned} \tag{26}$$

We see that $Q_2 + R_1 \ge 0$ and

$$R_1(Q_2+R_1)^\dagger(Q_2+R_1) = R_1 \tag{27}$$

hold because $\mathrm{Ker}\,(Q_2+R_1) \subseteq \mathrm{Ker}\,R_1$ holds. These conditions enable us to apply completing the square with respect to y_2 to the right hand side of (26). Then, (26) is bounded by

$$\begin{aligned}&\sum_{i=1}^2 \begin{bmatrix}u_i(\tau)\\ y_i(\tau)\end{bmatrix}^\top \begin{bmatrix}-R_i & I_m\\ I_m & -Q_i\end{bmatrix}\begin{bmatrix}u_i(\tau)\\ y_i(\tau)\end{bmatrix}\\ &\le \begin{bmatrix}w(\tau)\\ z(\tau)\end{bmatrix}^\top \begin{bmatrix}-R_1 & I_m\\ I_m & -Q_1-R_2\end{bmatrix}\begin{bmatrix}w(\tau)\\ z(\tau)\end{bmatrix}\\ &\quad+ \begin{bmatrix}w(\tau)\\ z(\tau)\end{bmatrix}^\top \begin{bmatrix}R_1\\ 0\end{bmatrix}(Q_2+R_1)^\dagger\begin{bmatrix}R_1\\ 0\end{bmatrix}^\top\begin{bmatrix}w(\tau)\\ z(\tau)\end{bmatrix}.\end{aligned}$$

Substituting (27) into $-R_1 + R_1(Q_2+R_1)^\dagger R_1$, we have that

$$\int_0^T \begin{bmatrix}w(\tau)\\ z(\tau)\end{bmatrix}^\top \begin{bmatrix}-R_{\rm FB} & I_m\\ I_m & -Q_{\rm FB}\end{bmatrix}\begin{bmatrix}w(\tau)\\ z(\tau)\end{bmatrix}d\tau \ge \rho_{\rm FB} \tag{28}$$

holds for all $T \in \mathbb{R}_+$, where $Q_{\rm FB}$ and $R_{\rm FB}$ are given by (8) and (9), i.e., $\Sigma_{\rm FB}(\Sigma_1,\Sigma_2) \in \mathcal{P}(Q_{\rm FB},R_{\rm FB})$ holds. In addition, applying (25) to $\Sigma_{\rm FB}(\Sigma_1,\Sigma_2) \in \mathcal{P}(Q_{\rm FB},R_{\rm FB})$, we show that $\mathcal{P}_{\rm FB} \subseteq \mathcal{P}(Q_{\rm FB},R_{\rm FB})$ holds. This completes the proof of Lemma 2.

D. Supplementary Lemma for Proof of Theorem 1

Lemma 3: Consider $Y \geq 0$ and $Z \geq 0$. Then, the following two statements are equivalent:

i) $\underline{\mathbb{V}}(Y)$ and $\operatorname{Ker} Z$ are disjoint.

ii) $\underline{\lambda}(Y+Z) > \underline{\lambda}(Y)$.

Proof of Lemma 3: (i $\Rightarrow$ ii) Any vector $q \in \underline{\mathbb{V}}(Y+Z)$ is decomposed as $q = q_1 + q_2$, where $q_1 \in \operatorname{Im} Z$ and $q_2 \in \operatorname{Ker} Z$. Then, we have

$$\begin{aligned} \underline{\lambda}(Y+Z) &= q^\top (Y+Z) q \\ &= (q_1+q_2)^\top Y (q_1+q_2) + q_1^\top Z q_1. \end{aligned} \tag{29}$$

First, suppose that $q_1 \neq 0$. Noting $(q_1+q_2)^\top Y(q_1+q_2) \geq \underline{\lambda}(Y)$ and $q_1^\top Z q_1 > 0$ hold, we have that $\underline{\lambda}(Y+Z) > \underline{\lambda}(Y)$ holds. Next, suppose that $q_1 = 0$, or in other words, $q \in \operatorname{Ker} Z$. Then, the right-hand side of (29) is reduced to $q_2^\top Y q_2$. From the statement i), we see that $q_2 \notin \underline{\mathbb{V}}(Y) \setminus \{0\}$ holds, which implies $q_2^\top Y q_2 > \underline{\lambda}(Y)$. Therefore, this shows that $\underline{\lambda}(Y+Z) > \underline{\lambda}(Y)$.

(ii $\Rightarrow$ i) We suppose that $\underline{\mathbb{V}}(Y)$ and $\operatorname{Ker} Z$ are not disjoint to derive a contradiction. In other words, there exists $v \in \underline{\mathbb{V}}(Y)$ such that $v \in \operatorname{Ker} Z \setminus \{0\}$. Then, $\underline{\lambda}(Y+Z) \leq v^\top (Y+Z) v = \underline{\lambda}(Y)$. This contradicts the statement ii). ■

E. Proof of Proposition 1

From the parameter transition (21), we see that, for any ℓ, all off-diagonal entries of $Q_{\mathrm{ent},\ell}$ do not change from those of Q. This implies that, for any ℓ, $Q_{\mathrm{ent},\ell} \in \mathbb{M}$ and $\mathcal{P}_{\mathrm{ent},\ell} \subseteq \mathcal{P}(Q_{\mathrm{ent},\ell}, 0)$ hold.

Next, we show that, for any ℓ, (22) holds if and only if Q is irreducible.

(Necessary condition for (22)) Suppose that $Q > 0$, $r_\ell > 0$, and (22) hold, and equivalently (23) holds. From Lemma 3, it follows that $\underline{\mathbb{V}}(Q)$ and $\operatorname{Ker} E_\ell$ are disjoint. Suppose that Q is reducible to derive a contradiction. Then, at some k, $\{Q\}_{kj} = 0$ holds for all $j \neq k$. Then, $v \in \underline{\mathbb{V}}(Q)$ has at least one zero entry. This contradicts that, for any ℓ, (22) holds.

(Sufficient condition for (22)) Suppose that Q is an irreducible M-matrix. Then, letting δ be the maximum value of the diagonal element of Q, we can decompose Q as

$$Q = \delta I_m - \Xi, \tag{30}$$

where $\Xi \in \mathbb{R}^{m \times m}$ is an irreducible non-negative matrix. From the Perron–Frobenius theorem [23], all entries of the eigenvector corresponding to $\overline{\lambda}(\Xi)$ are positive. Thus, from (30), all entries of $v \in \underline{\mathbb{V}}(Q) \setminus \{0\}$ are positive. Then, for any ℓ, $v \notin \operatorname{Ker} E_\ell$ holds. From Lemma 3, it follows that (23) holds. Therefore, it follows that for any ℓ, (22) holds. ■

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