Day-Ahead Scheduling for Supply-Demand-Storage Balancing –Model Predictive Generation with Interval Prediction of Photovoltaics–

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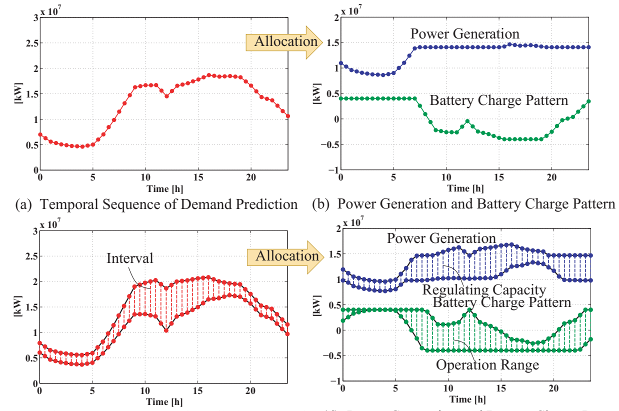
***Abstract*— Large-scale penetration of photovoltaic (PV) power generators and storage batteries is expected into the power system in Japan. To maintain the supply-demand balance with energy storage, the optimal power generation and the charge/discharge power of storage batteries can be determined in a manner of the model predictive control of generators. In view of this, this paper addresses a problem of the day-ahead scheduling for the supply-demand-storage balance with explicit consideration of the model predictive power generation. This scheduling is performed by using demand prediction, whose uncertainty is expressed in terms of interval prediction. Formulating the day-ahead scheduling problem as an interval-valued allocation problem, we give a solution to it by taking an approach based on the monotonicity analysis with respect to the optimal solution. Finally, the efficiency of the proposed method is verified through a numerical simulation, where we use an interval prediction of PV power generation constructed by experimental data.**



(a) Temporal Sequence of Demand Prediction (b) Power Generation and Battery Charge Pattern

(c) Temporal Sequence of Demand Prediction Intervals (d) Power Generation and Battery Charge Pattern

Fig. 1. Allocation Problem with Interval Demand Prediction.

I. INTRODUCTION

Recently, renewable energy gathers attention to address the issues on the global warming and the depletion of natural resources. The Japanese government has provided a road map, called PV2030+ [1], towards the penetration of renewable energy sources as typified by photovoltaic (PV) power generation, and thus large-scale penetration of storage batteries and PV generators is expected in near future.

In the following, we use the term "demand" to represent the amount of power obtained by subtracting the amount of PV power generation from the amount of power consumption by consumers. At the phase of power system online operation, by online monitoring the amount of demand, we determine the amounts of power generation and battery charge/discharge power by solving an allocation problem to optimize economical efficiency. However, because the power generators cannot start up instantaneously, it is necessary to make a offline day-ahead schedule to decide how many power generators should start up at each moment on the day of interest. If a profile of demand prediction is obtained as shown in Fig. 1(a), such a scheduling problem can be addressed as a day-ahead allocation problem, in which the profile of demand prediction is divided into those of power generation and battery charge/discharge power as shown in Fig. 1(b).

It should be noted that the demand prediction inevitably includes some uncertainty in practice since PV power generation depends on weather. Thus, it is necessary to make the day-ahead schedules while taking it into consideration. There are several methods of renewable energy prediction in which uncertainty is expressed as confidence intervals [2], including prediction profiles with a certain probability (e.g., 95%). In particular, we use a confidence interval for PV power generation prediction produced by the method proposed in [3]. In this method, a prediction model based on support vector regression produces an interval with a predetermined confidence level from meteorological data [4].

The profiles of demand prediction are supposed to vary within the sequence of intervals as shown in Fig. 1(c). In compliance with this, the offline day-ahead schedules of power generation and battery charge/discharge power are to be obtained as the sequences of intervals, whose determination can be formulated as an *interval-valued allocation problem*. This interval-valued allocation problem aims at finding the upper and lower limits of the optimal power generation and charge/discharge power shown by the lines with circles in Fig. 1(d).

However, the interval-valued optimization problem is not necessarily easy to solve, because it cannot be solved by finding the optimal value for a finite number of grid points in

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the parameter space. To tackle this difficulty of the problem, a constraint propagation technique can be used in conjunction with branch-and-branch algorithms [5], [6]. However, this requires the direct computation, thereby incurring large computation loads. Moreover, a conservative solution may be derived.

We have proposed a fundamental method based on the monotonicity analysis for the optimal solution in [7] as a first step. This paper continues upon the research of [7] and provides its generalized version in the sense that we formulate the interval-valued allocation problem on the basis of model predictive control of power generators, which allows us to ensure a kind of feasibility in the online operation. Online operation values of generators and storage batteries are not determined in intervals of them in [7]. In this paper, online operation values of generators and storage batteries are determined in a manner of model predictive control. In this generalized optimal problem, we need to consider the monotonicity of the optimal solution with respect to not only the variables for demand but also that for battery stored energy. In addition, we develop theoretical extensions that enable us to take account of constraints on storage batteries such as a battery capacity.

Finally, we clarify our contribution in comparison with existing studies on robust model predictive control considering some uncertainty. In a standard problem of robust model predictive control, where the uncertainty is modeled as an interval-valued or stochastic parameter, one optimal trajectory of the decision variable, such as an input signal, is determined so as to minimize an objective function that evaluates, e.g., its worst case or expectation value; see [8], [9], [10], [11], [12], [13] and references therein. In contrast, our interval-valued optimization problem aims at finding an interval set containing an infinite number of the optimal trajectories of the decision variable, each of which corresponds to each trajectory of an interval-valued parameter. The solution of this novel type of problems clarifies the lower and upper limits of input signals required to achieve the optimal control for each trajectory of an uncertain parameter, which has good compatibility with power system operation based on the online monitoring of demand. As for the other approaches, [14], [15], [16], [17] discuss day-ahead scheduling problems with the prediction uncertainty of renewable energy in manners of fuzzy optimization, stochastic optimization, and dynamic programming However, they do not consider utilizing the online operation of power systems.

The following notation is used in this paper. We denote the set of real numbers by $\mathbb{R}$, the n-dimensional unit matrix by I_n, the n-dimensional all-ones vector by $\mathbf{1}_n$, the n-dimensional all-zeros vector by $\mathbf{0}_n$, the cardinality of a set $\mathcal{I}$ by $|\mathcal{I}|$, the power set of a set $\mathcal{I}$ by $\mathfrak{P}(\mathcal{I})$, and the rank of a matrix A by $\mathrm{rank}(A)$. For natural number n, let $\mathbb{N}[n] := \{1, 2, \ldots, n\}$. We denote a matrix composed of column vectors of I_n corresponding to the indices $\mathcal{I} \subseteq \mathbb{N}[n]$ by $e^n_{\mathcal{I}} \in \mathbb{R}^{n\times|\mathcal{I}|}$. In particular, if not confusing, we omit its superscript n.

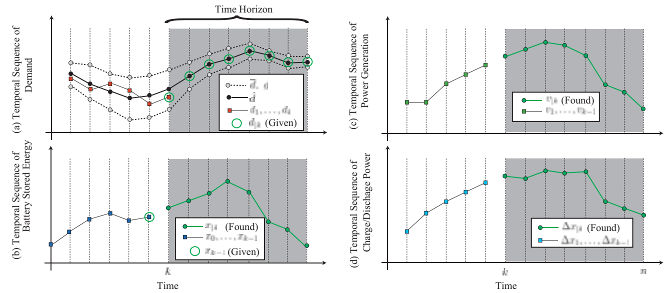

Fig. 2. Operation Problem at time k.

II. Problem Formulation

A. Online Operation Based on Model Predictive Control

In this section, we formulate an "offline" day-ahead allocation problem considering the model predictive control of power generators in power system operation. Dividing a day into n moments whose sampling period is $\Delta T := 24/n$ [hour], we determine appropriate power generation and battery charge/discharge cycles. In this subsection, we consider an "online" operation problem at time $k \in \mathbb{N}[n]$.

In this online operation problem, taking advantage of the online monitoring (updating) of demand, we consider determining the amount of power generation in a manner of model predictive control. In the power system operation at time k, suppose that we can use the information of

$$d_{|k} := \begin{bmatrix} d_k \\ \hat{d}_{k+1} \\ \hat{d}_{k+2} \\ \vdots \\ \hat{d}_n \end{bmatrix} \in \mathbb{R}^m, \quad m := n - k + 1 \tag{1}$$

and

$$x_{k-1} \in \mathbb{R} \tag{2}$$

where d_k denotes the amount of demand observed at time k, the sequence of $\hat{d}_i$ for $i \in \mathbb{N}[n]\backslash\mathbb{N}[k]$ denotes the profile of nominal demand prediction, and x_{k-1} denotes the amount of battery stored energy at time $k-1$. An illustration of $d_{|k}$ is given in Fig. 2 (a). In this notation, we describe the decision variables as

$$v_{|k} := \{v_{i|k}\} \in \mathbb{R}^m, \quad \Delta x_{|k} := \{\Delta x_{i|k}\} \in \mathbb{R}^m,$$
$$x_{|k} := \{x_{i|k}\} \in \mathbb{R}^m$$

where $v_{i|k}$, $\Delta x_{i|k}$, and $x_{i|k}$ denote the amounts of power generation, battery charge/discharge power, and battery stored energy at time $k+i-1$, respectively. Illustrations of $v_{|k}$, $\Delta x_{|k}$, and $x_{|k}$ are given in Figs. 2 (b), (c) and (d).

Next, we impose a set of constraint conditions on the decision variables. The inequality constraint for the upper and lower limits of charge/discharge power is given as

$$\Delta x_{\min}\mathbf{1}_m \leq \Delta x_{|k} \leq \Delta x_{\max}\mathbf{1}_m \tag{3}$$

where the constants $\Delta x_{\min} \in \mathbb{R}$ and $\Delta x_{\max} \in \mathbb{R}$ denote the lower and upper limits of charge/discharge power, respectively, which represent the inverter capacity. The inequality constraint for the upper and lower limits of battery stored energy is given as

$$x_{\min}\mathbf{1}_m \leq x_{|k} \leq x_{\max}\mathbf{1}_m \tag{4}$$

where the constants $x_{\min} \in \mathbb{R}$ and $x_{\max} \in \mathbb{R}$ denote the lower and upper limits of battery stored energy, respectively, which represent the battery capacity. The equality condition of $x_{m|k}$, i.e., the battery stored energy at termination time n, is given as

$$x_{m|k} = x_d \tag{5}$$

where the constant $x_d \in \mathbb{R}$ denotes the desired value at n. Furthermore, the equality condition representing the supply-demand balance is given as

$$\Delta x_{|k} = v_{|k} - d_{|k}. \tag{6}$$

Note that the temporal sequences of battery stored energy $x_{|k}$ can be represented by

$$x_{|k} = x_{k-1}\mathbf{1}_m + \Delta T M_m \Delta x_{|k} \tag{7}$$

where x_{k-1} denotes the battery stored energy at time $k-1$, $\Delta x_{|k}$ denotes the temporal sequence of charge/discharge power. $M_m \in \mathbb{R}^{m\times m}$ denotes the lower triangular matrix, whose (i,j)-element is given by

$$M_{m(i,j)} := \begin{cases} 1 & \text{if} \quad i \geq j \\ 0 & \text{otherwise} \end{cases}$$

Finally, the objective function is defined as

$$J_k(v_{|k}) := \Delta T \sum_{i=1}^{m} \left(a_0 + a_1 v_{i|k} + a_2 v_{i|k}^2 \right) \tag{8}$$

where $J_k(v_{|k})$ represents the fuel cost function of power generators, and the constants $a_0 \in \mathbb{R}, a_1 \in \mathbb{R}$, and $a_2 \in \mathbb{R}$ are appropriate positive values. Multiplying the objective function by $1/(2a_2\Delta T)$, $J_k(v_{|k})$ can be equivalently rewritten as

$$J_k(v_{|k}) := \frac{1}{2} v_{|k}^\top Q_{|k} v_{|k} - p_{|k}^\top v_{|k}, \tag{9}$$

where $Q_{|k}$ and $p_{|k}$ are given as

$$Q_{|k} := I_m \quad \text{and} \quad p_{|k} := -\frac{a_1}{2a_2}\mathbf{1}_m.$$

In this function, the constant term is omitted because it does not affect its minimizer.

Let us consider eliminating the redundant decision variable by using (6) and (7). Then, the operation problem at time k can be formulated as follows. First, we solve the quadratic programing problem

$$v_{|k}^* := \underset{v_{|k} \in \mathbb{R}^m}{\operatorname{argmin}} J_k(v_{|k}), \quad \text{s.t.} \begin{cases} A_{\text{in}|k} v_{|k} \leq b_{\text{in}|k} \\ A_{\text{eq}|k} v_{|k} = b_{\text{eq}|k} \end{cases} \tag{10}$$

where $v_{|k}^*$ denotes the optimal values of $v_{|k}$, $J_k(v_{|k})$ is defined as in (9), and $A_{\text{in}|k}$, $b_{\text{in}|k}$, $A_{\text{eq}|k}$ and $b_{\text{eq}|k}$ are defined as

$$A_{\text{in}|k} := \begin{bmatrix} I_m \\ -I_m \\ M_m \\ -M_m \end{bmatrix}, \tag{11}$$

$$b_{\text{in}|k} := \begin{bmatrix} \Delta x_{\max}\mathbf{1}_m + d_{|k} \\ -(\Delta x_{\min}\mathbf{1}_m + d_{|k}) \\ \frac{1}{\Delta T}(x_{\max} - x_{k-1})\mathbf{1}_m + M_m d_{|k} \\ -\{\frac{1}{\Delta T}(x_{\min} - x_{k-1})\mathbf{1}_m + M_m d_{|k}\} \end{bmatrix}, \tag{12}$$

$$A_{\text{eq}|k} := \mathbf{1}_m^\top, \tag{13}$$

$$b_{\text{eq}|k} := \frac{1}{\Delta T}(x_d - x_{k-1}) + \mathbf{1}_m^\top d_{|k}. \tag{14}$$

Next, by using $v_{|k}^*$, the optimal values of $\Delta x_{|k}$ and $x_{|k}$ are calculated as

$$\Delta x_{|k}^* := v_{|k}^* - d_{|k}, \tag{15}$$

$$x_{|k}^* := x_{k-1}\mathbf{1}_m + \Delta T M_m \Delta x_{|k}^*. \tag{16}$$

Finally, we determine the amounts of power generation, battery charge/discharge power, and battery stored energy at time k as $v_{1|k}^*$, $\Delta x_{1|k}^*$ and $x_{|1k}^*$, respectively.

B. Offline Day-Ahead Scheduling Problem Based on Interval Demand Prediction

In this subsection, on the basis of online operation in a manner of model predictive control in (10), we formulate an offline day-ahead scheduling problem for power generation and battery charge/discharge power. In this problem, d_k in (1) and x_{k-1} in (2) are dealt with as interval-valued parameters. We suppose that the nominal demand profile $\hat{d}_i$, the interval prediction $[\underline{d}_k, \overline{d}_k] \subset \mathbb{R}$ at each $k \in \mathbb{N}[n]$, and the initial battery stored energy $x_0 \in \mathbb{R}$ are given in advance. Then, let us consider the following day-ahead scheduling problem:

Problem 1: Consider the quadratic programming problem (10). Given $[\underline{d}_k, \overline{d}_k] \subset \mathbb{R}$ for each $k \in \mathbb{N}[n]$ and $x_0 \in \mathbb{R}$, define

$$\mathcal{V}_{|k}^* := \left\{ v_{|k}^*(d_k, x_{k-1}) \mid (d_k, x_{k-1}) \in \{[\underline{d}_k, \overline{d}_k] \times [\underline{x}_{k-1}, \overline{x}_{k-1}]\} \right\}$$

$$\Delta\mathcal{X}_{|k}^* := \left\{ \Delta x_{|k}^*(d_k, x_{k-1}) \mid (d_k, x_{k-1}) \in \{[\underline{d}_k, \overline{d}_k] \times [\underline{x}_{k-1}, \overline{x}_{k-1}]\} \right\}$$

$$\mathcal{X}_{|k}^* := \left\{ x_{|k}^*(d_k, x_{k-1}) \mid (d_k, x_{k-1}) \in \{[\underline{d}_k, \overline{d}_k] \times [\underline{x}_{k-1}, \overline{x}_{k-1}]\} \right\}$$

where $\underline{x}_{k-1}$ and $\overline{x}_{k-1}$ are defined as

$$\underline{x}_{k-1} := \begin{cases} x_0 & \text{if} \quad k=1 \\ \underline{x}_{k-1}^* & \text{otherwise} \end{cases}, \quad \overline{x}_{k-1} := \begin{cases} x_0 & \text{if} \quad k=1 \\ \overline{x}_{k-1}^* & \text{otherwise.} \end{cases}$$

Then, find

$$\overline{v}_{1|k}^*, \quad \underline{v}_{1|k}^*, \quad \overline{\Delta x}_{1|k}^*, \quad \underline{\Delta x}_{1|k}^*, \quad \overline{x}_{1|k}^*, \quad \underline{x}_{1|k}^*$$

where $\overline{v}_{1|k}^*$ and $\underline{v}_{1|k}^*$ denote the maximum and minimum values of the first elements of $v_{|k}^*$ for any $v_{|k}^* \in \mathcal{V}_{|k}^*$, i.e. the upper and lower bounds of the first elements of set $\mathcal{V}_{|k}^*$, and $\overline{\Delta x}_{1|k}^*, \underline{\Delta x}_{1|k}^*, \overline{x}_{1|k}^*$, and $\underline{x}_{1|k}^*$are defined as in the same manner. □

Note that the lower and upper limits of the optimal power generation, i.e., $\underline{v}^*_{1|k}(d_k, x_{k-1})$ and $\overline{v}^*_{1|k}(d_k, x_{k-1})$, can be used to determine how many generators should start up in advance.

III. Solution of Scheduling Problem Based on Monotonicity

A. Monotonicity of Solution of Quadratic Programming Problem

We first introduce the following notion of monotonicity:

Definition 1: Let $\mathcal{D} := [\underline{d}, \overline{d}] \subset \mathbb{R}^\nu$. A function $f : \mathcal{D} \to \mathbb{R}^N$ is said to be σ-*monotone* with respect to d, if, for any $d^{(1)}, d^{(2)} \in \mathcal{D}$ such that $d^{(1)}_j > d^{(2)}_j$ and $d^{(1)}_{\backslash j} = d^{(2)}_{\backslash j}$, there exists a constant matrix $\sigma \in \{-1, 1\}^{N\times\nu}$ such that

$$\sigma_{i,j}\left(f_i(d^{(1)}) - f_i(d^{(2)})\right) \geq 0, \quad \forall i \in \mathbb{N}[N], j \in \mathbb{N}[\nu]$$

where d_j denotes the jth element of d, $d_{\backslash j}$ denotes the subvector of d constructed by eliminating its jth element, $\sigma_{i,j}$ denotes the (i,j)-element of σ, and f_i denotes the ith element of f. □

We consider the maximum and minimum values of ith element of the function $f : \mathcal{D} \to \mathbb{R}^N$ for $\mathcal{D} := [\underline{d}, \overline{d}] \subset \mathbb{R}^\nu$. If the function f is σ-monotone with respect to d, the maximum and minimum values of the ith element of f can be represented as

$$\overline{f_i(d)} := \max\{f_i(d) \mid d \in \mathcal{D}\} = f_i(\overline{d}^{(i)})$$
$$\underline{f_i(d)} := \min\{f_i(d) \mid d \in \mathcal{D}\} = f_i(\underline{d}^{(i)})$$

where the jth element of $\overline{d}^{(i)}$ and $\underline{d}^{(i)}$ are defined as

$$\overline{d}^{(i)}_j := \sigma_{i,j} \max\{\sigma_{i,j}\underline{d}_j, \sigma_{i,j}\overline{d}_j\}$$
$$\underline{d}^{(i)}_j := \sigma_{i,j} \min\{\sigma_{i,j}\underline{d}_j, \sigma_{i,j}\overline{d}_j\}.$$

Next, we consider the parametrized quadratic programming problem given by

$$v^*(d) := \operatorname*{argmin}_{v\in\mathbb{R}^N} J(v) \quad \text{s.t.} \quad \begin{cases} A_{\rm in}v \leq b_{\rm in}(d) \\ A_{\rm eq}v = b_{\rm eq}(d) \end{cases} \tag{17}$$

where

$$A_{\rm in} \in \mathbb{R}^{M\times N},\ A_{\rm eq} \in \mathbb{R}^{R\times N},\ b_{\rm in}(d) \in \mathbb{R}^M,\ b_{\rm eq}(d) \in \mathbb{R}^R$$

and

$$J(v) := \frac{1}{2}v^\top Qv - p^\top v$$

with a positive definite matrix $Q = Q^\top \in \mathbb{R}^{N\times N}$ and a vector $p \in \mathbb{R}^N$. If $b_{\rm in}(d)$ and $b_{\rm eq}(d)$ are continuous functions of d, then $v^*(d)$ is a continuous function of d.

It is difficult to analyze the monotonicity of $v^*(d)$ in a direct way since $v^*(d)$ is defined by using "argmin". Thus, we introduce the solution candidate and analyze monotonicity of the candidate. For any $\mathcal{I} \in \mathfrak{P}(\mathbb{N}[M])$ such that

$$\text{rank}(A_\mathcal{I}) = |\mathcal{I}| + R \tag{18}$$

where M and R denote the dimensions of $b_{\rm in}(d)$ and $b_{\rm eq}(d)$, the solution candidate $v^c(\mathcal{I}; d)$ is defined as

$$v^c(\mathcal{I}; d) := \begin{bmatrix} I_n & \mathbf{0}_{n\times(|\mathcal{I}|+R)} \end{bmatrix} \begin{bmatrix} Q & A_\mathcal{I}^\top \\ A_\mathcal{I} & \mathbf{0}_{(|\mathcal{I}|+R)\times(|\mathcal{I}|+R)} \end{bmatrix}^{-1} \begin{bmatrix} p \\ b_\mathcal{I}(d) \end{bmatrix} \tag{19}$$

where

$$A_\mathcal{I} := \begin{bmatrix} e_\mathcal{I}^\top A_{\rm in} \\ A_{\rm eq} \end{bmatrix} \in \mathbb{R}^{(|\mathcal{I}|+R)\times N} \tag{20}$$

$$b_\mathcal{I}(d) := \begin{bmatrix} e_\mathcal{I}^\top b_{\rm in}(d) \\ b_{\rm eq}(d) \end{bmatrix} \in \mathbb{R}^{(|\mathcal{I}|+R)}. \tag{21}$$

Then, there exists $\mathcal{I}^*$ such that

$$v^*(d) = v^c(\mathcal{I}^*; d)$$

(See [7]). If the solution candidate $v^c(\mathcal{I}; d)$ is σ-monotone with respect to d for "any" $\mathcal{I}$, then $v^*(d)$ is σ-monotone with respect to d. Thus, by using $v^c(\mathcal{I}; d)$, we can analyze the monotonicity of $v^*(d)$ in an indirect manner.

B. Solution to Scheduling Problem

First, we analyze monotonicity of the solution $v^*_{|k}$ in (10). The solution $v^*_{|k}$ is a function of $d_k \in [\underline{d}_k, \overline{d}_k] \subset \mathbb{R}$ and $x_{k-1} \in [\underline{x}_{k-1}, \overline{x}_{k-1}] \subset \mathbb{R}$. In view of this, we show that $v^*_{|k}$ is monotone with respect to d_k and x_{k-1}. For simplicity of notation, we omit the subscript $_{|k}$ in the following, if not confusing.

By using (19), the solution candidate for the solution v^* can be described as

$$v^c(\mathcal{I}) = A_\mathcal{I}^\top(A_\mathcal{I}A_\mathcal{I}^\top)^{-1}b_\mathcal{I} + \{I_m - A_\mathcal{I}^\top(A_\mathcal{I}A_\mathcal{I}^\top)^{-1}A_\mathcal{I}\}p,$$

where $A_\mathcal{I}$ is defined as in (20) and $b_\mathcal{I}$ is defined as in (21). The elements of $A_\mathcal{I}$, $A_{\rm in}$ and $A_{\rm eq}$, are defined as in (11) and (13), respectively. The elements of $b_\mathcal{I}$, $b_{\rm in}$ and $b_{\rm eq}$, are defined as in (12) and (14), respectively. Furthermore, the partial derivatives of $v^c(\mathcal{I})$ with respect to $d_{|k}$ and x_{k-1} can be calculated as

$$\frac{\partial v^c(\mathcal{I})}{\partial d_{|k}} = A_\mathcal{I}^\top(A_\mathcal{I}A_\mathcal{I}^\top)^{-1}A_\mathcal{I} \tag{22}$$

$$\frac{\partial v^c(\mathcal{I})}{\partial x_{k-1}} = A_\mathcal{I}^\top(A_\mathcal{I}A_\mathcal{I}^\top)^{-1}\frac{\partial b_\mathcal{I}}{\partial x_{k-1}}. \tag{23}$$

Then, the following lemma shows that $v^c(\mathcal{I})$ is monotone with respect to $d_{|k}$.

Lemma 1: For any $\mathcal{I}$ such that (18), $v^c(\mathcal{I})$ is σ-monotone with respect to $d_{|k}$ for $\sigma = \mathbf{1}_{m\times m}$. Furthermore, all elements of $\frac{\partial v^c(\mathcal{I})}{\partial d_{|k}} \in \mathbb{R}^{m\times m}$ are nonnegative and less than or equal to 1. □

Furthermore, the following lemma shows that $v^c(\mathcal{I})$ is monotone with respect to x_{k-1}.

Lemma 2: For any $\mathcal{I}$ such that (18), $v^c(\mathcal{I})$ is σ-monotone with respect to x_{k-1} for $\sigma = -\mathbf{1}_{m\times 1}$. Furthermore, all elements of $\frac{\partial v^c(\mathcal{I})}{\partial x_{k-1}} \in \mathbb{R}^{m\times 1}$ are nonpositive and greater than or equal to $-\frac{1}{\Delta T}$. □

By using *Lemma 1 and 2*, the solution v^* is shown to be monotone with respect to $d_{|k}$ and x_{k-1}. Next, we analyze monotonicity of Δx^* in (15) and x^* in (16). They are the functions of d_k and x_{k-1} as well as v^*. To prove their monotonicity, we consider monotonicity of Δx^c and x^c defined as

$$\Delta x^c(\mathcal{I}) := v^c(\mathcal{I}) - d_{|k} \tag{24}$$
$$x^c(\mathcal{I}) := x_{k-1}\mathbf{1}_m + \Delta T M_m \Delta x^c(\mathcal{I}). \tag{25}$$

Then, the following lemma shows that $\Delta x^c(\mathcal{I})$ and $x^c(\mathcal{I})$ are monotone with respect to $d_{|k}$ and x_{k-1}.

Lemma 3: For any $\mathcal{I}$ such that (18),

- $\Delta x^c(\mathcal{I})$ is $\sigma_{\Delta x/d}$-monotone with respect to $d_{|k}$,
- $\Delta x^c(\mathcal{I})$ is $\sigma_{\Delta x/x}$-monotone with respect to x_{k-1},
- $x^c(\mathcal{I})$ is $\sigma_{x/d}$-monotone with respect to $d_{|k}$, and
- $x^c(\mathcal{I})$ is $\sigma_{x/x}$-monotone with respect to x_{k-1}

with $\sigma_{\Delta x/d}$, $\sigma_{\Delta x/x}$, $\sigma_{x/d}$, and $\sigma_{x/x}$ defined as

$$\sigma_{\Delta x/d(i,j)} := \begin{cases} -1 & \text{if} \quad i=j \\ 1 & \text{otherwise} \end{cases}, \quad \sigma_{\Delta x/x(i)} := -1$$
$$\sigma_{x/d(i,j)} := \begin{cases} -1 & \text{if} \quad i\geq j \\ 1 & \text{otherwise} \end{cases}, \quad \sigma_{x/x(i)} := 1$$

where $\sigma_{\Delta x/d(i,j)}$ denotes the (i,j)-element of $\sigma_{\Delta x/d}$, $\sigma_{\Delta x/x(i)}$ denotes the ith element of $\sigma_{\Delta x/x}$, and $\sigma_{x/d(i,j)}$ and $\sigma_{x/x(i)}$ are defined as in the same manner. □

By using *Lemma 3*, Δx^* and x^* are shown to be monotone with respect to $d_{|k}$ and x_{k-1}. The following theorem gives the solution to *Problem 1*.

Theorem 1: The solution of *Problem 1* is given as

$$\overline{v}^*_{1|k} = v^*_{1|k}(\overline{d}_k, \underline{x}_{k-1}), \qquad \underline{v_{1|k}}^* = v^*_{1|k}(\underline{d}_k, \overline{x}_{k-1})$$
$$\overline{\Delta x}^*_{1|k} = \Delta x^*_{1|k}(\underline{d}_k, \underline{x}_{k-1}), \quad \underline{\Delta x}^*_{1|k} = \Delta x^*_{1|k}(\overline{d}_k, \overline{x}_{k-1})$$
$$\overline{x}^*_{1|k} = x^*_{1|k}(\underline{d}_k, \overline{x}_{k-1}), \qquad \underline{x}^*_{1|k} = x^*_{1|k}(\overline{d}_k, \underline{x}_{k-1}).$$

□

Proof: From the results of *Lemma 1*, *2*, and *3*, $v^*(d_k, x_{k-1})$, $\Delta x^*(d_k, x_{k-1})$, and $x^*(d_k, x_{k-1})$ are monotone with respect to d_k, x_{k-1}. Then, the claim follows. ■

By using Theorem 1, the upper and lower limits of the optimal generation profile can be obtained by solving the quadratic programming with $2n$.

IV. Numerical Simulation

In this section, we show the efficiency of the proposed method for scheduling of power generation and battery charge/discharge cycles. We consider the supply-demand balance in Tokyo area having 19 million consumers, where three million consumers have storage batteries.

The demand prediction is made on the basis of an actual power consumption data $q \in \mathbb{R}^{24}$ on May 30, 2010, and PV prediction data on the same day, which corresponds to the nominal profile of PV power prediction $\hat{p} \in \mathbb{R}^{24}$ and its upper and lower limits $\overline{p} \in \mathbb{R}^{24}, \underline{p} \in \mathbb{R}^{24}$. The upper and

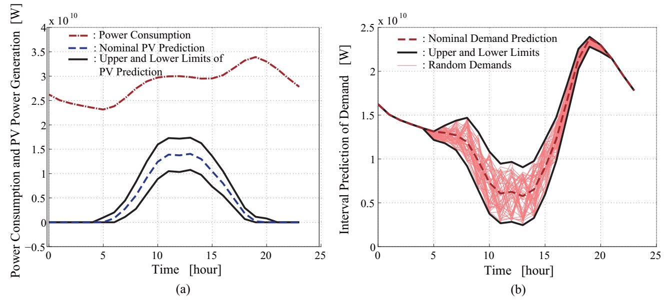

Fig. 3. (a) Power consumption and PV power prediction, (b) Interval Prediction of demand

TABLE I
PARAMETERS

parameter	numerical value	unit
$\Delta x_{\min}$	3000	[MW]
$\Delta x_{\max}$	-3000	[MW]
$x_{\min}$	0	[MWh]
$x_{\max}$	15000	[MWh]
x_0	0	[MWh]
x_d	0	[MWh]
a_0	3.16×10^5	[JPY/h]
a_1	4.60×10^{-3}	[JPY/Wh]
a_2	1.05×10^{-12}	[JPY/W^{2}h]

lower limits of demand prediction $\overline{d} \in \mathbb{R}^{24}$, $\underline{d} \in \mathbb{R}^{24}$ and the nominal profile of demand prediction $\hat{d} \in \mathbb{R}^{24}$ are calculated as $q - \overline{p}$, $q - \underline{p}$ and $q - \hat{p}$, respectively. Since the prediction uncertainty of power consumption is much lower than that of PV, that of power consumption is zero in the simulation.

The actual power consumption data with one hour sampling is available from [18]. The data of PV power prediction is made based on the statistics of the past data. It should be noted that we scale the PV power prediction data so that its peak value becomes 30GW, which corresponds to the situation where 10 million consumers have the solar panels of 3kW complying with [1].

The profiles of the power consumption and the PV prediction interval are shown in Fig. 3(a), where the power consumption, the nominal PV prediction, the upper and lower limits of PV prediction are shown by the dashed-dotted line, the dotted line and the solid lines, respectively.

From them, we obtain the demand prediction interval as shown in Fig. 3(b), where we subtract 10 GW corresponding to the amount of power generation by basis generators, such as nuclear plants. Here, for the simulation of operation on the day of interest, we produce 50 trajectories of demand prediction shown by the thin solid lines, which are randomly produced within the prediction interval.

We set the parameters in (2)–(8) as in Table I. The parameters a_0, a_1, a_2 are given according to [7]. Under this setting, for the demand prediction interval in Fig. 3(b), we perform the day-ahead scheduling. The results are shown in Fig. 4(a), (b), and (c), which are the optimal profiles of power generation, battery charge/discharge power, and battery stored energy, respectively. In each figure, the thick solid lines denote the upper and lower limits of the corresponding

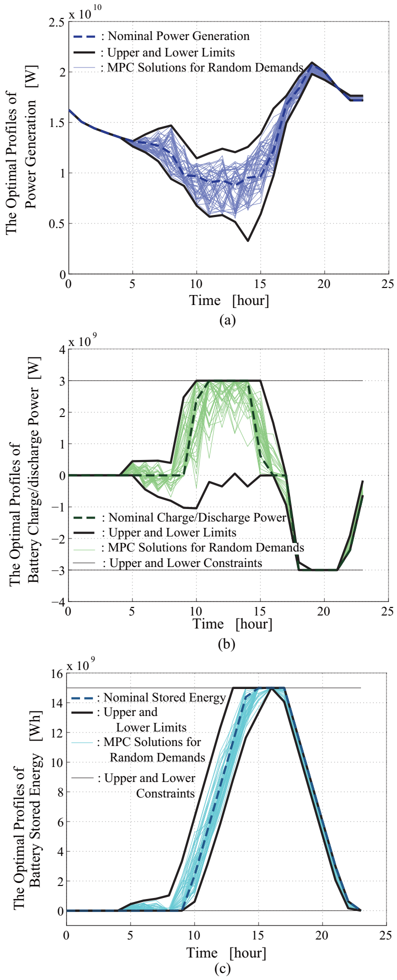

Fig. 4. (a) The Optimal Profiles of Power Generation, (b) The Optimal Profiles of Battery Charge/Discharge Power, (c) The Optimal Profiles of Battery Stored Energy

values, the dotted line denotes the solution profile of model predictive control for the nominal demand prediction, and the thin solid lines represent the solution profile of model predictive control for the randomly produced demand profiles in Fig.3(b).

Thin solid lines in each figure does not reach the upper and lower limits computed via monotonicity analysis (monotonicity method). This implies that exactly estimating the upper and lower limits by using random demand profile (Monte Carlo method) is difficult. In contrast, the proposed method provides its upper and lower limits appropriately.

V. Concluding Remarks

This paper has addressed a problem of the day-ahead scheduling for the supply-demand-storage balance with explicit consideration of the model predictive power generation. We use interval prediction to express the prediction uncertainty and formulate the problem of the day-ahead scheduling as an interval-valued allocation problem. In this interval-valued allocation problem, we suppose that the amount of power generation is determined in a manner of model predictive control that is compatible with the online monitoring of demand at the phase of operation. Furthermore, we have given a solution to the problem by using an approach based on the monotonicity analysis with respect to the optimal solution. Finally, we have shown that the proposed method provides the upper and lower limits for power generation and charge/discharge power appropriately by using the numerical simulations. A generalization to multiple generators as well as a consideration of the charge/discharge efficiency of storage batteries are currently under investigation.

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