Planning of Optimal Daily Power Generation Tolerating Prediction Uncertainty of Demand and Photovoltaics

Masakazu Koike [*,†] **Takayuki Ishizaki** [*,†] **Yuzuru Ueda** [**,†]
Taisuke Masuta [***,†] **Takashi Ozeki** [***,†] **Nacim Ramdani** [****]
Tomonori Sadamoto [*,†] **Jun-ichi Imura** [*,†]

[*] *Graduate School of Information Science and Engineering, Tokyo Institute of Technology 2-12-1, Ookayama, Meguro, Tokyo, 152–8552, Japan (Tel: +81-3-5734-2646; e-mail: { koike, ishizaki, sadamoto, imura } @cyb.mei.titech.ac.jp)*
[**] *Graduate School of Science and Engineering, Tokyo Institute of Technology 2-12-1, Ookayama, Meguro, Tokyo, 152–8552, Japan (e-mail: ueda.y.ae@m.titech.ac.jp)*
[***] *National Institute of Advanced Industrial Science and Technology 1-2-1, Namiki, Tsukuba, Ibaraki, 305–8564, Japan (e-mail: { taisuke.masuta, takashi.oozeki} @aist.go.jp)*
[****] *Universite d'Orleans, PRISME,63 av. de Lattre de Tassigny, 18020 Bourges, France (e-mail: Nacim.Ramdani@univ-orleans.fr)*
[†] *CREST, Japan Science and Technology Agency 4–1–8, Honcho, Kawaguchi, Saitama, 332–0012, Japan*

Abstract: With increased attention of renewable energy, large-scale installation of photovoltaic (PV) generation and electricity storage is expected to be installed into the power system in Japan. In this situation, we need to keep supply-demand balance by systematically using traditional power generation systems as well as the PV generation and storage equipment. Towards this balancing, a number of prediction methods for PV generation and demand have been developed in literature. However, prediction-based balancing is not necessarily easy. This is because the prediction of PV generation and demand inevitably includes some uncertainty. Against this background, we formulate a problem to plan battery charge pattern while minimizing the fuel cost of generators with explicit consideration of prediction uncertainty. In this problem, given as interval quadratic programming, the prediction uncertainty is described as a parameter in constraint condition. Furthermore, we propose a method to find a solution to this problem from the viewpoint of monotonicity analysis. Finally, by numerical analysis based on this problem and its solution method, we discuss the relation between the minimal regulating capacity and the required battery charge/discharge pattern to tolerate a given amount of prediction uncertainty.

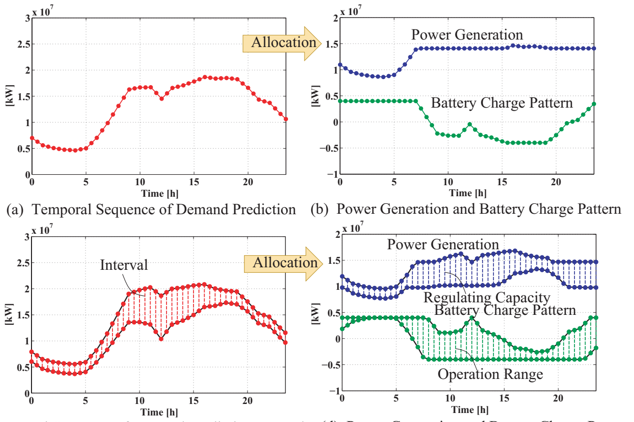
Keywords: Photovoltaic Power Generation, Prediction Uncertainty, Battery Charge Pattern Planning, Regulating Capacity

1. INTRODUCTION

Recently, the global warming and the depletion of natural resources have been a serious problem in energy environment. In view of this, renewable energy as typified by photovoltaic (PV) power generation has been gathering attention to reduce carbon dioxide as well as to achieve sustainable energy consumption. Actually in Japan, a large number of PV power generators and storage batteries are expected to be installed into power systems in the near future [T. Masuta et al. 2012], [R. Komiyama et al. 2011].

In this situation, we are required to operate a power system that includes the traditional generators as well as PV generators and storage batteries while keeping the supply-demand balance of the power. If a PV/demand prediction, which is a *net* value of demand prediction defined as the difference of the demand prediction and the PV power output prediction, is exactly available, one can schedule the power generation and the battery charge pattern by solving an optimization problem. This optimization problem can be formulated as an allocation problem of a sequence of the PV/demand prediction to those of the generated power and battery charge/discharge power (see Fig. 1 (a) and (b)) with the minimization of an energy cost function.

However, in practice, the PV/demand prediction inevitably includes some uncertainty. In view of this, we express the uncertainty of PV/demand prediction as a temporal sequence of prediction values that can vary within a



(a) Temporal Sequence of Demand Prediction (b) Power Generation and Battery Charge Pattern

(c) Temporal Sequence of Demand Prediction Intervals (d) Power Generation and Battery Charge Pattern

Fig. 1. Allocation Problem.

sequence of intervals; see Fig. 1 (c). These intervals have known bounds but the actual distribution of the uncertainty within these bounds is unknown. This unknown but bounded error thus gathers both systematic and random variations and uncertainty. To keep the supply-demand balance for all possible PV/demand predictions within the intervals, it is important to estimate how wide the ranges of generator power and battery charge power should be. More specifically, to clarify the ranges of generator power and battery charge power to be required, we need to find the upper and lower limits of them shown by the lines with circles in Fig. 1 (d). It should be noted that the sequence of power generation intervals coincides with the sequence of regulating capacity that can tolerate a given amount of prediction uncertainty.

In this paper, we find the upper and lower limits of generator power and battery charge power that minimize a quadratic fuel cost function of generators. Owing to the fact that the PV/demand prediction within an interval can be regarded as a continuous parameter within an vector-valued interval, we can formulate the problem as an *interval quadratic programming*.

It should be noted that to give a solution to the interval quadratic programming is not necessarily easy. This is because we are required to solve the quadratic programming for all possible parameters, i.e., infinite many parameters, to derive the optimal solutions as a function of the continuous parameter.

To tackle this difficult problem, we use an interval analysis technique; see [E. Hansen et al. 2003], [L. Jaulin et al. 2001], [Q.G. Lin et al 2008], [Y. Zhu et al 2012], [L. He et al 2009]. Interval analysis literature focuses mainly on global optimization, i.e. finding the global maximum/minimum point of a multimodal multivariable function, or on constraint satisfaction problems, i.e. covering the feasible solution set of conjunctions of equality and inequality constraints. Most techniques use constraint propagation, and some of them also use time consuming branch-and-bound algorithms [E. Hansen et al. 2003], [L. Jaulin et al. 2001]. By using these method directly, we can solve the interval quadratic programming. However, the method would require partitioning and computing directly with interval of real numbers, and hence potentially huge computation costs are required. To the best of our knowledge, there is no method based on the interval analysis for solving an interval quadratic programming in an effective way.

To overcome this difficulty, we propose a method to find the upper and lower limits of solutions, which is based the monotonicity analysis. This method has the advantage to exactly find the upper and lower limits by a finite number of operations. Finally, by a numerical analysis, we discuss the relation between the minimal regulating capacity and the required charge pattern to tolerate a given PV/demand prediction interval.

2. PROBLEM FORMULATION

2.1 Quadratic Programming for Optimal Power Generation Planning

In this section, based on uncertain prediction of PV generation and demand, we formulate a problem to minimize the fuel cost of generators as a quadratic programming. Here, we consider obtaining an one-day plan of power generation as well as charge and discharge pattern of storage batteries. In general, the one-day plan is calculated by using unit commitment. However, for simplicity, we do not consider the start-up cost of generators.

Dividing a day into n moments, we denote the temporal sequences of predicted PV power generation and demand by $p \in \mathbb{R}^n$ and $q \in \mathbb{R}^n$, respectively. Using these symbols, we define a *net* amount of demand prediction as $d := q - p$. Furthermore, we describe its time sequence as

$$d = \{d_i\} \in \mathbb{R}^n \tag{1}$$

where d_i denotes the ith element of d. In what follows, we refer to d in (1) just as a *PV/demand prediction*.

For this PV/demand prediction, we keep a supply-demand balance using the power of generators as well as the charge and discharge power of storage batteries. The total power of all generators and the total charge and discharge power of all storage batteries are described by

$$v = \{v_i\} \in \mathbb{R}^n, \quad \Delta x = \{\Delta x_i\} \in \mathbb{R}^n \tag{2}$$

on which we impose the inequality constraint as

$$\begin{cases} v_{\min} \le v_i \le v_{\max} \\ \Delta x_{\min} \le \Delta x_i \le \Delta x_{\max}, \end{cases} \quad i \in \{1, \ldots, n\}. \tag{3}$$

In (3), the constants $v_{\min} \in \mathbb{R}$, $v_{\max} \in \mathbb{R}$, $\Delta x_{\min} \in \mathbb{R}$ and $\Delta x_{\max} \in \mathbb{R}$ represent the lower and upper limits of v_i and Δx_i. In this notation, the supply-demand balance at the ith moment is represented by

$$\Delta x_i = v_i - d_i, \quad i \in \{1, \ldots, n\}. \tag{4}$$

Furthermore, we denote the total energy of the batteries by

$$x = \{x_i\} \in \mathbb{R}^n, \quad x_i := x_0 + \sum_{j=1}^{i} \Delta x_j \tag{5}$$

where $x_0 \in \mathbb{R}$ denotes the initial value of the total energy, and we impose the equality constraint on this battery energy as

$$x_n = x_0 + x_{\mathrm{d}} \tag{6}$$

where $x_n \in \mathbb{R}$ denotes the total energy at the termination time, and $x_{\mathrm{d}} \in \mathbb{R}$ denotes a desired energy to be charged.

We define the fuel cost function of the generators by

$$J(v) := \sum_{i=1}^{n} a_0 + a_1 v_i + a_2 v_i^2 \tag{7}$$

where $a_0 \in \mathbb{R}$, $a_1 \in \mathbb{R}$ and $a_2 \in \mathbb{R}$ are the nonnegative coefficients. Then, by representing Δx with v and d based on (4), the optimal power generation plan that minimizes the fuel cost of generators is given by

$$v^*(d) := \arg\min_{v \in \mathbb{R}^n} J(v) \tag{8}$$

where the inequality constraint in (3) can be rewritten as

$$C_{\mathrm{in}}(v; d) : \begin{cases} v_{\min} \leq v_i \leq v_{\max} \\ \Delta x_{\min} + d_i \leq v_i \leq \Delta x_{\max} + d_i, \end{cases} \tag{9}$$

and the equality constraint in (6) can be rewritten as

$$C_{\mathrm{eq}}(v; d) : \sum_{i=1}^{n} v_i = \sum_{i=1}^{n} d_i + x_{\mathrm{d}}. \tag{10}$$

In addition, as being compatible with (4), the optimal plan of charge pattern is given by

$$\Delta x^*(d) := v^*(d) - d, \tag{11}$$

and the optimal temporal sequence of battery energy is given by

$$x^*(d) = \{x_i^*(d)\} \in \mathbb{R}^n, \quad x_i^*(d) := x_0 + \sum_{j=1}^{i} \Delta x_j^*(d). \tag{12}$$

2.2 Interval Quadratic Programming

In this subsection, we formulate a problem to plan the power generation and the battery charge pattern explicitly taking into account the uncertainty of PV/demand prediction. More specifically, we regard d in the quadratic programming as a continuous parameter that can vary within a vector-valued interval.

Let $[\underline{d}, \overline{d}] \subseteq \mathbb{R}^n$ denote an interval of the PV/demand prediction d in (1), and we refer to $[\underline{d}, \overline{d}] \subseteq \mathbb{R}^n$ as a *prediction interval*. Furthermore, we refer to the central value

$$\mathbf{d} := \frac{\underline{d} + \overline{d}}{2} \in \mathbb{R}^n \tag{13}$$

as a *nominal PV/demand prediction*.

As shown in (8) and (11), the optimal power generation plan v^* and the optimal charge and discharge pattern Δx^* depend on the PV/demand prediction d varying within the interval $[\underline{d}, \overline{d}]$. In view of this, we refer to the quadratic programming in (8) as an *interval quadratic programming*. For this quadratic programming, we formulate a problem to find the upper and lower limits of v^* and Δx^* as follows:

Problem 1. Consider an interval quadratic programming in (8). Let a prediction interval $[\underline{d}, \overline{d}] \subseteq \mathbb{R}^n$ be given. For a parameter $d \in [\underline{d}, \overline{d}]$, define the inequality and equality constraints by

$$C_{\mathrm{in}}(v; d), \quad C_{\mathrm{eq}}(v; \mathbf{d}) \tag{14}$$

where C_{in} and C_{eq} are defined as in (9) and (10), and the nominal PV/demand prediction $\mathbf{d}$ is defined as in (13). Furthermore, define

$$\begin{aligned} \mathcal{V}^* &:= \{v^*(d) : d \in [\underline{d}, \overline{d}]\} \\ \Delta\mathcal{X}^* &:= \{\Delta x^*(d) : d \in [\underline{d}, \overline{d}]\} \end{aligned} \tag{15}$$

where v^* and Δx^* are defined as in (8) and (11). Find

$$\overline{v}^* = \{\overline{v}_i^*\} \in \mathbb{R}^n, \quad \underline{v}^* = \{\underline{v}_i^*\} \in \mathbb{R}^n \tag{16}$$

and

$$\overline{\Delta x}^* = \{\overline{\Delta x}_i^*\} \in \mathbb{R}^n, \quad \underline{\Delta x}^* = \{\underline{\Delta x}_i^*\} \in \mathbb{R}^n \tag{17}$$

where $\overline{v}_i^*$ and $\underline{v}_i^*$ indicate the maximum and minimum value of the ith element of v^* for any $v^* \in \mathcal{V}^*$, and $\overline{\Delta x}_i^*$ and $\underline{\Delta x}_i^*$ are defined as in the same manner. ■

In Problem 1, we formulate a problem to find the upper and lower limits of all possible solutions v^* and Δx^* with respect to any $d \in [\underline{d}, \overline{d}]$. It should be noted that, in this problem, the equality constraint in (14) is given by using the nominal PV/demand prediction $\mathbf{d}$, which is a constant vector. This is because, if the equality constraint depends on the parameter d, the resulting interval quadratic programming does not possess *monotonicity*, which is a key notion to solve Problem 1; see Section 3 for details.

The upper and lower limits of v^* can be regarded as the regulating capacity that can cover any PV/demand prediction $d \in [\underline{d}, \overline{d}]$. To reduce the number of generators while guaranteeing stable power supply, it is important to find the minimum regulating capacity that can cover any uncertain demand prediction.

Note, however, that the solution of Problem 1 is not necessarily easy to obtain in general. This is because, in order to derive the optimal solution as a function of the parameter d, we are required to solve the interval quadratic programming for all possible parameters, i.e., infinite many parameters. This fact implies that a finite number of solutions for a fixed $d \in [\underline{d}, \overline{d}]$ does not give the exact lower and upper limits in (16) and (17).

In theory, the direct application of interval arithmetics, i.e. the extension of real algebraic operations to intervals, may be used for computing the upper and lower limits of (16) and (17). In practice, the computed limits may be conservative since interval arithmetics relies on over-approximation, and may also requires large computation time. Therefore, we investigate in next section a more effective way to solve the interval quadratic programming using interval analysis without interval arithmetics, by relying on a monotonicity analysis. The upper and lower limits of the image of an interval by a monotone function can be computed directly by using only the upper and lower limits of the interval.

3. MONOTONICITY ANALYSIS

In this section, we analyze the interval quadratic programming from a viewpoint of monotonicity. To this end, the following notion of monotonicity is introduced:

Definition 1. The interval quadratic programming in Problem 1 is said to be *monotone* with respect to d if, for any

$i \in \{1, \dots, n\}$ and $j \in \{1, \dots, n\}$, there exist constants $\sigma_{i,j}^{(v)} \in \{-1, 1\}$ and $\sigma_{i,j}^{(\Delta x)} \in \{-1, 1\}$ such that

$$\sigma_{i,j}^{(v)} \frac{\partial v_i^*(d)}{\partial d_j} \geq 0, \quad \sigma_{i,j}^{(\Delta x)} \frac{\partial \Delta x_i^*(d)}{\partial d_j} \geq 0 \quad \forall d \in [\underline{d}, \overline{d}] \tag{18}$$

where v_i^* and Δx_i^* denote the ith elements of v^* and Δx^* defined as in (8) and (11), respectively. ■

The monotonicity of the interval quadratic programming is defined as the existence of $\sigma_{i,j}^{(v)} \in \{-1, 1\}$ and $\sigma_{i,j}^{(\Delta x)} \in \{-1, 1\}$ such that (18), which means that the signs of $\partial v_i^* / \partial d_j$ and $\partial \Delta x_i^* / \partial d_j$ are invariant. Note that, if the interval quadratic programming is monotone with respect to d, then the upper and lower limits of v_i^* are exactly given as $\overline{v}_i^* = v_i^*(\overline{d}^{(i)})$, $\underline{v}_i^* = v_i^*(\underline{d}^{(i)})$ where the jth elements of $\overline{d}^{(i)} \in \mathbb{R}^n$ and $\underline{d}^{(i)} \in \mathbb{R}^n$ are defined by

$$\begin{cases} \overline{d}_j^{(i)} := \sigma_{i,j}^{(v)} \max \left\{ \sigma_{i,j}^{(v)} \underline{d}_j, \sigma_{i,j}^{(v)} \overline{d}_j \right\} \\ \underline{d}_j^{(i)} := \sigma_{i,j}^{(v)} \min \left\{ \sigma_{i,j}^{(v)} \underline{d}_j, \sigma_{i,j}^{(v)} \overline{d}_j \right\}. \end{cases}$$

This fact implies that the solutions v^* with a finite number of d exactly give the upper and lower limits of v^*. Obviously, we can obtain the upper and lower limits of Δx^* in the same manner.

As for the monotonicity of the interval quadratic programming in Problem 1, we can prove the following theorem:

Theorem 1. Consider the interval quadratic programming in Problem 1. If

$$\sigma_{i,j}^{(v)} = \begin{cases} 1, & i = j, \\ -1, & i \neq j, \end{cases} \quad \sigma_{i,j}^{(\Delta x)} = -1, \tag{19}$$

then (18) follows for any $i \in \{1, \dots, n\}$ and $j \in \{1, \dots, n\}$.

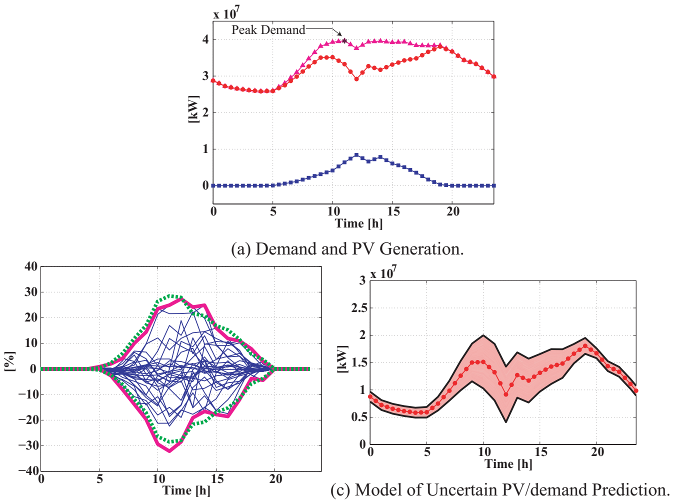
This theorem shows that the interval quadratic programming in Problem 1 possesses the monotonicity characterized by (19). Note that the upper and lower limits of Δx^* can be obtained by solving the quadratic programming with $\underline{d}$ and $\overline{d}$. Consequently, the solution of Problem 1 is exactly obtained by solving the quadratic programming with $2n + 2$ kinds of d.

Recall that, in Problem 1, the equality constraint in (14) is given by the nominal PV/demand prediction $\mathbf{d}$ in (13). If the equality constraint also depends on d, the interval quadratic programming does not possess the monotonicity in general. Thus, in this case, we cannot obtain the upper and lower limits in (16) and (17) by a finite number of calculation; recall the last of Section 2.2. The efficiency of analyses based on this monotonicity is demonstrated by numerical simulations in Section 4.

4. NUMERICAL SIMULATION

4.1 Prediction of PV Generation and Demand

In this section, we show the efficiency of the proposed method to plan power generation and battery charge pattern. We consider the supply-demand balance in Tokyo area having 19 million demanders, where five million demanders have PV generators and three million demanders have storage batteries.



(a) Demand and PV Generation.

(b) Prediction Error of PV Generation.

(c) Model of Uncertain PV/demand Prediction.

Fig. 2. Demand and PV Generation, Prediction Error of PV Generation and Model of Uncertain PV/demand Prediction for Cloudy Day.

We suppose that the upper and lower limits of generated power in (3) are 50 GW and 25 GW, respectively. The coefficients of the fuel cost function in (7) are given as $a_0 = 3.16 \times 10^5$ JPY/h?$a_1 = 4.60 \times 10^{-3}$ JPY/Wh and $a_2 = 1.05 \times 10^{-12}$ JPY/W^{2}h [T. Masuta et al. 2013].

In the following numerical simulation, we use an actual demand data on June 9, 2010 [Tokyo Electric Power Co.,Inc. 2012] as a demand prediction, and use a PV generation data in the same day, which is produced by the solar radiation data [Japan Meteorological Business Support Center 2013], as a PV generation prediction. In Fig. 2(a), the solid lines with triangles, with squares and with circles represent the demand prediction data, the PV generation prediction data, and the PV/demand prediction data, respectively.

The PV generation data is calculated as follows: First, the data of vertical quantity of total solar radiation is multiplied by a PV rating capacity as well as the system output coefficient 0.8. Second, the obtained data is divided by the standard solar irradiance 1.0 kW/m^2. Finally, we scale it so that the maximum PV power output is 15 GW, which corresponds to a target amount of the PV power output by 2030 in Japan.

In Fig. 2(b), the area enclosed by the two dashed lines represent the temporal sequence of PV generation prediction intervals. The dashed lines are calculated as follows: First, we prepare two sets of data for certain 30 days. One is the set of actual data for 30 days that resemble the data on June 9, 2010; the other is the set of prediction data for the same 30 days. Second, as the difference of data sets between the actual and prediction data, we obtain the data set of prediction error that are shown by the thin solid lines in Fig. 2(b). Furthermore, we show the envelope curves of the prediction error set as the thick solid lines. Finally, we obtain the dashed lines by adjusting the envelope curves so that the dashed lines are symmetric with respect to the horizontal axis.

In addition, we also consider the temporal sequence of demand prediction intervals that is given as a band corresponding to ±5% with respect to the peak demand, shown by the mark of $*$ on the solid line with triangles in Fig. 2(a). By adding the temporal sequences of the PV generation prediction intervals and demand prediction intervals, we obtain the sequence of the PV/demand prediction interval shown in Fig. 2(c), where the central solid line with circles represents the nominal PV/demand prediction $\mathbf{d}$ in (13). Note that, in this figure, we have already subtracted 20 GW to be covered by basis generators, such as nuclear plants and so forth.

4.2 Evaluation Indices for Power Generation Planning

We introduce some quantitative evaluation indices for power generation planning. We suppose that a desirable plan of power generation accomplishes the following conditions as much as possible:

(i) The maximum number of operated generators is minimized.
(ii) The change rate for the number of operated generators is minimized.
(iii) The number of generators to tolerate the prediction uncertainty is minimized.

From the viewpoints of **(i)**, **(ii)** and **(iii)**, we evaluate a resultant power generation plan.

The optimal solution for the nominal PV/demand prediction, is expressed as $\mathbf{v}^* = v^*(\mathbf{d})$ where v^* is defined as in (8). We refer to $\mathbf{v}^*$ as the *nominal power generation plan.* The upper and lower limits of the optimal power generation plan are defined as in (16).

Let us mathematically describe the evaluation indices **(i)**, **(ii)** and **(iii)** as

$$\begin{cases} W_1 := \max\limits_{i\in\{1,\dots,48\}} \mathbf{v}_i^* \ , \quad W_2 := \sum\limits_{i=1}^{47} |\mathbf{v}_{i+1}^* - \mathbf{v}_i^*| \\ W_3 := \sum\limits_{i=1}^{48} \dfrac{\overline{v}_i^* - \underline{v}_i^*}{\mathbf{v}_i^*} \end{cases} \tag{20}$$

where $\mathbf{v}_i^*$ denotes the ith element of $\mathbf{v}^*$. If the value of W_1 is small, the maximum number of operated generators is small. Similarly, if the value of W_2 is small, we do not require high readiness of generators, and if the value of W_3 is small, the number of generators to tolerate the prediction uncertainty is small.

4.3 Planing of Power Generation and Battery Charge Pattern

The simulation results are shown in Figs. 3(a), (b) and (c) where we use the PV/demand prediction defined in Section 4.1. In these figures, we show the results in the cases where we vary the upper and lower limits of the GW capacity of batteries as **Case 1**: ±2.5 GW, **Case 2**: ±4.0 GW and **Case 3**: ±5.5 GW. In each figure, the first subfigure shows the resultant plans of power generation v^* and battery charge pattern Δx^*, defined as in (8) and (11), and the second one shows the corresponding temporal sequence of battery energy x, defined as in (12). The solid lines with circles represent the nominal power generation plan and the corresponding nominal battery charge plan and the temporal sequence of battery energy. The thick solid lines show the upper and lower limits of all possible solutions for any $d \in [\underline{d}, \overline{d}]$. These lines can be obtained in a few seconds by solving the quadratic programming with $2n + 2$ kinds of d because the problem in this simulation possesses the monotonicity. The indication by the color density is explained in Section 4.4.

We can see from each first subfigure that, as the GW capacity of batteries becomes larger, the difference between the upper and lower limits of the generated power becomes smaller. Furthermore, as shown in each second subfigure, the nominal temporal sequence of the battery energy at the termination time returns to its initial value. This is ensured by setting x_d in (6) to zero for the equality constraint in (14). Moreover, the difference between the upper and lower limits of the battery energy increase with the lapse of time. This is because the battery energy is defined by the temporal integration of charged power.

Next, in Table 1, we show the values of the evaluation indices in (20) for **Case 1**, **Case 2** and **Case 3**. In this table, we normalize the the evaluation index values so that $W_1 = W_2 = W_3 = 1$ holds if no storage batteries are installed. In this case, the temporal sequence of generated power must be identical to that of the PV/demand prediction; namely, it follows that $\mathbf{d} = \mathbf{v}^*$, $\overline{d} = \overline{v}^*$, $\underline{d} = \underline{v}^*$. We can see form Table 1 that every evaluation index decreases in the order from **Case 1** to **Case 3**. This fact indicates that batteries possessing a larger GW capacity is more effective to provide a desirable power generation plan, from a viewpoint of **(i)**, **(ii)** and **(iii)**.

Note that the value of W_3 in **Case 1** is greater than one. This means that we require a regulating capacity that is larger than one in the case where no storage batteries are installed. This may be caused by the fact $\sigma_{i,j}^{(v)} \neq \sigma_{i,j}^{(\Delta x)}$ shown in Theorem 1, which means that a set of demand corresponding to the upper and lower limits of the optimal power generation plan is not identical to that of the optimal battery charge pattern plan. In conclusion, we see that the storage batteries with larger GW capacity are required to tolerate a larger amount of uncertainty.

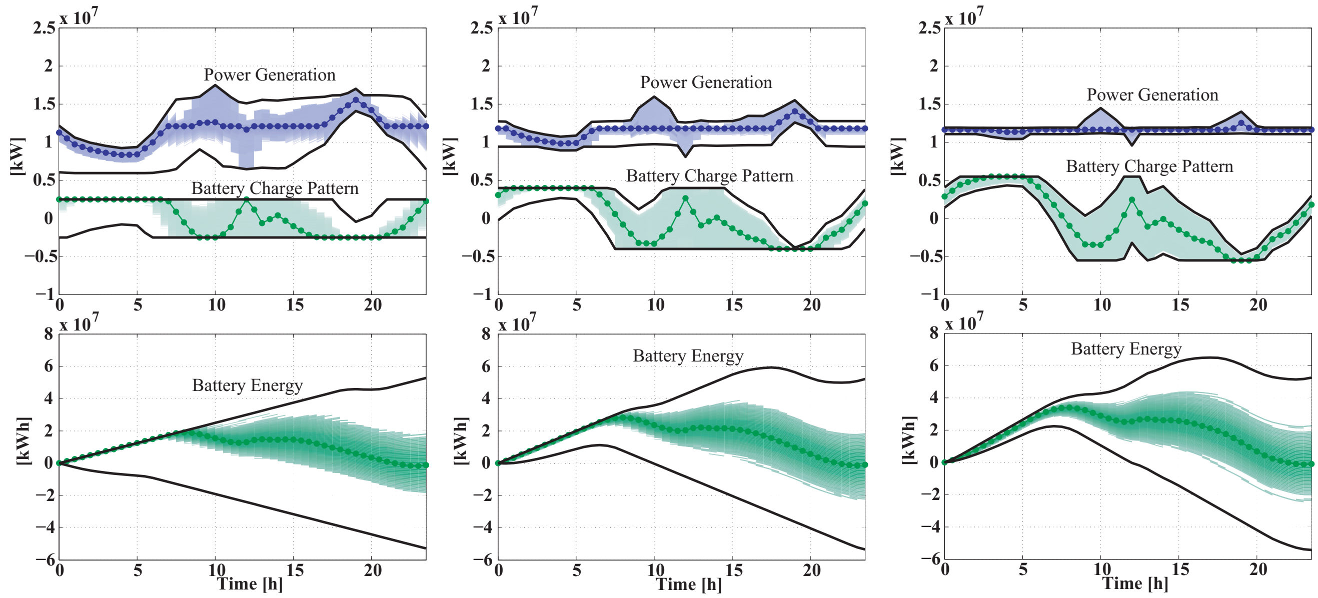
4.4 Discussion on Total Battery Capacity

We investigate a total battery capacity that is required to tolerate prediction uncertainty. The temporal sequence of battery energy is shown in the each second subfigure of Figs. 3(a), (b) and (c). We can see from these figures that the differences between the peak values of the upper and lower limits are 108 GWh, 113 GWh and 119 GWh, respectively. To tolerate the worst case, we need to have storage batteries whose total GWh capacity is about 120 GWh; see **Case 3**.

Next, to investigate an occurrence rate of the worst case, we calculate the optimal plans of power generation and battery charge pattern for 10000 PV/demand predictions randomly chosen from $[\underline{d}, \overline{d}]$. In Figs. 3(a), (b) and (c), the

Table 1. Normalized Values of Indices for Cloudy Day.

	W_1	W_2	W_3
Case 1	0.86	0.42	1.41
Case 2	0.78	0.23	0.72
Case 3	0.70	0.07	0.27



(a) Case 1. (b) Case 2. (c) Case 3.

Fig. 3. Simulation Results in from **Case 1** to **Case 3**.

color density at each point within the intervals reflects the number of trajectories passing through around the corresponding point.

From each first subfigure, we can see that the trajectories of power generation and battery charge pattern pass through uniformly in the intervals by the upper and lower limits. On the other hand, in each second subfigure, the trajectories of battery energy gather around its nominal trajectory. This indicates that the variance of the battery energy trajectories is less than that of the generated power and the battery charge power. This result comes from the fact that battery energy is defined as the temporal integration of charged power. Therefore, we see that the total battery capacity should be considered with the consideration of the GWh capacity both in the worst and common cases.

5. CONCLUSION

In this paper, we have formulated a problem to plan power generation and battery charge pattern as an interval quadratic programming. Furthermore, we have proposed a solution method to this problem. The solution of the interval quadratic programming can be regarded as the regulating capacity that can tolerate any uncertain demand prediction. Moreover, we have provided a number of simulation results to verify the efficiency of the proposed method for power generation planning. By these numerical simulations, we have found the following facts:

- Storage batteries having larger GW capacity are more effective to minimize the regulating capacity.
- If prediction uncertainty is relatively large compared with the GW capacity of batteries, we require regulating capacity that is larger than one in the case where no storage batteries are installed.

A generalization to multiple generators as well as a consideration of PV generation surplus are currently under investigation.

REFERENCES

T. Masuta, A. Murata and T. Oozeki. Basic study on economic-load dispatching control based on photovoltaic generation forecasting. *The papers of Joint Technical Meeting on Power Engineering, Power System Engineering and Semiconductor Power Converter, IEE Japan*, PE-13-023, PSE-13-039, SPC-13-059, 2013 (in Japanese)

T. Masuta and A. Yokoyama. Supplementary load frequency control by use of a number of both electric vehicles and heat pump water heaters. *IEEE Transactions on Smart Grid*, Vol.3, No.3, pp.1253–1262, 2012

R. Komiyama, S. Shibata, Y. Nakamura and Y. Fujii. Analysis on possible introduction of PV system considering output power fluctuation and battery technology employing optimal power generation mix model. *IEEJ Transactions on Electronics, Information and Systems*, Vol.131, No.10, pp.1705–1714, 2011 (in Japanese)

E. Hansen and G. W. Walster. Global optimization using interval analysis. *revised and expanded. Vol. 264. CRC Press*, pp.15–42, 2003

L. Jaulin, M. Kieffer, O. Didrit and E. Walter. Applied interval analysis: with examples in parameter and state estimation, robust control and robotics. *Springer*, pp.17–25, 2001

Q. G. Lin and G. H. Huang. IPEM: an interval-parameter energy systems planning model. *Energy Sources, Part A*, Vol.30, No.14-15, pp.1382–1399, 2008

Y. Zhu, Y. P. Li and G. H. Huang. Planning municipal-scale energy systems under functional interval uncertainties. *Renewable Energy*, Vol.39, No.1, pp.71–84, 2012

L. He, G. H. Huang, G. Zeng and H. Lu. An interval mixed-integer semi-infinite programming method for municipal solid waste management. *Journal of the Air & Waste Management Association*, Vol.59, No.2, pp.236-246, 2009

Tokyo Electric Power Co.,Inc. Download electricity demand data. http://www.tepco.co.jp/forecast/(2012)

Japan Meteorological Business Support Center. http://www.jmbsc.or.jp/ (2013)