Planning of Optimal Daily Power Generation Tolerating Prediction Uncertainty of Demand and Photovoltaics

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Abstract: With increased attention of renewable energy, large-scale installation of photovoltaic (PV) generation and electricity storage is expected to be installed into the power system in Japan. In this situation, we need to keep supply-demand balance by systematically using traditional power generation systems as well as the PV generation and storage equipment. Towards this balancing, a number of prediction methods for PV generation and demand have been developed in literature. However, prediction-based balancing is not necessarily easy. This is because the prediction of PV generation and demand inevitably includes some uncertainty. Against this background, we formulate a problem to plan battery charge pattern while minimizing the fuel cost of generators with explicit consideration of prediction uncertainty. In this problem, given as interval quadratic programming, the prediction uncertainty is described as a parameter in constraint condition. Furthermore, we propose a method to find a solution to this problem from the viewpoint of monotonicity analysis. Finally, by numerical analysis based on this problem and its solution method, we discuss the relation between the minimal regulating capacity and the required battery charge/discharge pattern to tolerate a given amount of prediction uncertainty.

Keywords: Photovoltaic Power Generation, Prediction Uncertainty, Battery Charge Pattern Planning, Regulating Capacity

1. INTRODUCTION

Recently, the global warming and the depletion of natural resources have been a serious problem in energy environment. In view of this, renewable energy as typified by photovoltaic (PV) power generation has been gathering attention to reduce carbon dioxide as well as to achieve sustainable energy consumption. Actually in Japan, a large number of PV power generators and storage batteries are expected to be installed into power systems in the near future [T. Masuta et al. 2012], [R. Komiyama et al. 2011].

In this situation, we are required to operate a power system that includes the traditional generators as well as PV generators and storage batteries while keeping the supply-demand balance of the power. If a PV/demand prediction, which is a net value of demand prediction defined as the difference of the demand prediction and the PV power output prediction, is exactly available, one can schedule the power generation and the battery charge pattern by solving an optimization problem. This optimization problem can be formulated as an allocation problem of a sequence of the PV/demand prediction to those of the generated power and battery charge/discharge power (see Fig. 1 (a) and (b)) with the minimization of an energy cost function.

However, in practice, the PV/demand prediction inevitably includes some uncertainty. In view of this, we express the uncertainty of PV/demand prediction as a temporal sequence of prediction values that can vary within a
sequence of intervals; see Fig. 1 (c). These intervals have known bounds but the actual distribution of the uncertainty within these bounds is unknown. This unknown but bounded error thus gathers both systematic and random variations and uncertainty. To keep the supply-demand balance for all possible PV/demand predictions within the intervals, it is important to estimate how wide the ranges of generator power and battery charge power should be. More specifically, to clarify the ranges of generator power and battery charge power to be required, we need to find the upper and lower limits of them shown by the lines with circles in Fig. 1 (d). It should be noted that the sequence of power generation intervals coincides with the sequence of regulating capacity that can tolerate a given amount of prediction uncertainty.

In this paper, we find the upper and lower limits of generator power and battery charge power that minimize a quadratic fuel cost function of generators. Owing to the fact that the PV/demand prediction within an interval can be regarded as a continuous parameter within an vector-valued interval, we can formulate the problem as an interval quadratic programming.

It should be noted that to give a solution to the interval quadratic programming is not necessarily easy. This is because we are required to solve the quadratic programming for all possible parameters, i.e., infinite many parameters, to derive the optimal solutions as a function of the continuous parameter.

To tackle this difficult problem, we use an interval analysis technique; see [E. Hansen et al. 2003], [L. Jaulin et al. 2001], [Q.G. Lin et al. 2008], [Y. Zhu et al. 2012], [L. He et al 2009]. Interval analysis literature focuses mainly on global optimization, i.e. finding the global maximum/minimum point of a multimodal multivariable function, or on constraint satisfaction problems, i.e. covering the feasible solution set of conjunctions of equality and inequality constraints. Most techniques use constraint propagation, and some of them also use time consuming branch-and-bound algorithms [E. Hansen et al. 2003], [L. Jaulin et al. 2001]. By using these method directly, we can solve the interval quadratic programming. However, the method would require partitioning and computing directly with interval of real numbers, and hence potentially huge computation costs are required. To the best of our knowledge, there is no method based on the interval analysis for solving an interval quadratic programming in an effective way.

To overcome this difficulty, we propose a method to find the upper and lower limits of solutions, which is based the monotonicity analysis. This method has the advantage to exactly find the upper and lower limits by a finite number of operations. Finally, by a numerical analysis, we discuss the relation between the minimal regulating capacity and the required charge pattern to tolerate a given PV/demand prediction interval.

2. PROBLEM FORMULATION

2.1 Quadratic Programming for Optimal Power Generation Planning

In this section, based on uncertain prediction of PV generation and demand, we formulate a problem to minimize the fuel cost of generators as a quadratic programming. Here, we consider obtaining an one-day plan of power generation as well as charge and discharge pattern of storage batteries. In general, the one-day plan is calculated by using unit commitment. However, for simplicity, we do not consider the start-up cost of generators.

Dividing a day into \( n \) moments, we denote the temporal sequences of predicted PV power generation and demand by \( p \in \mathbb{R}^n \) and \( q \in \mathbb{R}^n \), respectively. Using these symbols, we define a net amount of demand prediction as \( d := q - p \).

Furthermore, we describe its time sequence as

\[
\mathbf{d} = \{d_i\} \in \mathbb{R}^n
\]  

(1)

where \( d_i \) denotes the \( i \)th element of \( d \). In what follows, we refer to \( d \) in (1) just as a PV/demand prediction.

For this PV/demand prediction, we keep a supply-demand balance using the power of generators as well as the charge and discharge power of storage batteries. The total power of all generators and the total charge and discharge power of all storage batteries are described by

\[
\mathbf{v} = \{v_i\} \in \mathbb{R}^n, \quad \Delta \mathbf{x} = \{\Delta x_i\} \in \mathbb{R}^n
\]  

(2)

on which we impose the inequality constraint as

\[
\begin{align*}
\min v & \leq v_i \leq \max v \\
\min x & \leq \xi_i \leq \max x
\end{align*}, \quad i \in \{1, \ldots, n\}.
\]  

(3)

In (3), the constants \( v_{\min} \in \mathbb{R}, v_{\max} \in \mathbb{R}, \Delta x_{\min} \in \mathbb{R} \) and \( \Delta x_{\max} \in \mathbb{R} \) represent the lower and upper limits of \( v_i \) and \( \Delta x_i \). In this notation, the supply-demand balance at the \( i \)th moment is represented by

\[
\Delta x_i = v_i - d_i, \quad i \in \{1, \ldots, n\}.
\]  

(4)

Furthermore, we denote the total energy of the batteries by

\[
x = \{x_i\} \in \mathbb{R}^n, \quad x_i := x_0 + \sum_{j=1}^{i} \Delta x_j
\]  

(5)

where \( x_0 \in \mathbb{R} \) denotes the initial value of the total energy, and we impose the equality constraint on this battery energy as

\[
x_n = x_0 + x_d
\]  

(6)

where \( x_0 \in \mathbb{R} \) denotes the total energy at the termination time, and \( x_d \in \mathbb{R} \) denotes a desired energy to be charged.

Fig. 1. Allocation Problem.
We define the fuel cost function of the generators by
\[ J(v) := \sum_{i=1}^{n} a_0 + a_1 v_i + a_2 v_i^2 \] (7)
where \( a_0 \in \mathbb{R}, a_1 \in \mathbb{R} \) and \( a_2 \in \mathbb{R} \) are the nonnegative coefficients. Then, by representing \( \Delta x \) with \( v \) and \( d \) based on (4), the optimal power generation plan that minimizes the fuel cost of generators is given by
\[ v^*(d) := \arg \min_{v \in \mathbb{R}^n} J(v) \] (8)
where the inequality constraint in (3) can be rewritten as
\[ C_{in}(v; d) := \left\{ v_{\min} \leq v_i \leq v_{\max}, \Delta x_{\min} + d_i \leq v_i \leq \Delta x_{\max} + d_i, \right\} \] (9)
and the equality constraint in (6) can be rewritten as
\[ C_{eq}(v; d) := \sum_{i=1}^{n} v_i = \sum_{i=1}^{n} d_i + x_d. \] (10)
In addition, as being compatible with (4), the optimal plan of charge pattern is given by
\[ \Delta x^*(d) := v^*(d) - d, \] (11)
and the optimal temporal sequence of battery energy is given by
\[ x^*(d) = \{x^*_i(d)\} \in \mathbb{R}^n, \quad x^*_i(d) := x_0 + \sum_{j=1}^{i} \Delta x^*_j(d). \] (12)

2.2 Interval Quadratic Programming

In this subsection, we formulate a problem to plan the power generation and the battery charge pattern explicitly taking into account the uncertainty of PV/demand prediction. More specifically, we regard \( d \) in the quadratic programming as a continuous parameter that can vary within a vector-valued interval.

Let \( [d, \tilde{d}] \subseteq \mathbb{R}^n \) denote an interval of the PV/demand prediction \( d \) in (1), and we refer to \( [d, \tilde{d}] \subseteq \mathbb{R}^n \) as a prediction interval. Furthermore, we refer to the central value \( d := \frac{d + \tilde{d}}{2} \in \mathbb{R}^n \) (13) as a nominal PV/demand prediction.

As shown in (8) and (11), the optimal power generation plan \( v^* \) and the optimal charge and discharge pattern \( \Delta x^* \) depend on the PV/demand prediction \( d \) varying within the interval \( [d, \tilde{d}] \). In view of this, we refer to the quadratic programming in (8) as an interval quadratic programming. For this quadratic programming, we formulate a problem to find the upper and lower limits of \( v^* \) and \( \Delta x^* \) as follows:

**Problem 1.** Consider an interval quadratic programming in (8). Let a prediction interval \( [d, \tilde{d}] \subseteq \mathbb{R}^n \) be given. For a parameter \( d \in [d, \tilde{d}] \), define the inequality and equality constraints by
\[ C_{in}(v; d), \quad C_{eq}(v; d) \] (14)
where \( C_{in} \) and \( C_{eq} \) are defined as in (9) and (10), and the nominal PV/demand prediction \( d \) is defined as in (13). Furthermore, define
\[ V^* := \{v^*(d) : d \in [d, \tilde{d}]\} \]
\[ \Delta x^* := \{\Delta x^*(d) : d \in [d, \tilde{d}]\} \] (15)
where \( v^* \) and \( \Delta x^* \) are defined as in (8) and (11). Find
\[ \Psi^* = \{\Psi^*_i\} \in \mathbb{R}^n, \quad \Psi^* = \{\Psi^*_i\} \in \mathbb{R}^n \] (16)
and
\[ \Delta x^*_i = \{\Delta x^*_i\} \subseteq \mathbb{R}^n, \quad \Delta x^*_i = \{\Delta x^*_i\} \subseteq \mathbb{R}^n \] (17)
where \( \Psi^*_i \) and \( \Delta x^*_i \) indicate the maximum and minimum value of the \( i \)th element of \( v^* \) for any \( v^* \in V^* \), and \( \Delta x^*_i \) and \( \Delta x^*_i \) are defined as in the same manner.

In Problem 1, we formulate a problem to find the upper and lower limits of all possible solutions \( v^* \) and \( \Delta x^* \) with respect to any \( d \in [d, \tilde{d}] \). It should be noted that, in this problem, the equality constraint in (14) is given by using the nominal PV/demand prediction \( d \), which is a constant vector. This is because, if the equality constraint depends on the parameter \( d \), the resulting interval quadratic programming does not possess monotonicity, which is a key notion to solve Problem 1; see Section 3 for details.

The upper and lower limits of \( v^* \) can be regarded as the regulating capacity that can cover any PV/demand prediction \( d \in [d, \tilde{d}] \). To reduce the number of generators while guaranteeing stable power supply, it is important to find the minimum regulating capacity that can cover any uncertain demand prediction.

Note, however, that the solution of Problem 1 is not necessarily easy to obtain in general. This is because, in order to derive the optimal solution as a function of the parameter \( d \), we are required to solve the interval quadratic programming for all possible parameters, i.e., infinite many parameters. This fact implies that a finite number of solutions for a fixed \( d \in [d, \tilde{d}] \) does not give the exact lower and upper limits in (16) and (17).

In theory, the direct application of interval arithmetics, i.e. the extension of real algebraic operations to intervals, may be used for computing the upper and lower limits of (16) and (17). In practice, the computed limits may be conservative since interval arithmetics relies on over-approximation, and may also requires large computation time. Therefore, we investigate in next section a more effective way to solve the interval quadratic programming using interval analysis without interval arithmetics, by relying on a monotonicity analysis. The upper and lower limits of the image of an interval by a monotone function can be computed directly by using only the upper and lower limits of the interval.

3. MONOTONICITY ANALYSIS

In this section, we analyze the interval quadratic programming from a viewpoint of monotonicity. To this end, the following notion of monotonicity is introduced:

**Definition 1.** The interval quadratic programming in Problem 1 is said to be monotone with respect to \( d \) if, for any
\(i \in \{1, \ldots, n\} \text{ and } j \in \{1, \ldots, n\}\), there exist constants
\(\sigma_{i,j}^{(v)} \in \{-1, 1\}\) and \(\sigma_{i,j}^{(dx)} \in \{-1, 1\}\) such that
\[
\sigma_{i,j}^{(v)} \frac{\partial v_i^*(d)}{\partial d_j} \geq 0, \quad \sigma_{i,j}^{(dx)} \frac{\partial \Delta x_i^*(d)}{\partial d_j} \geq 0 \quad \forall d \in [\underline{d}, \overline{d}] \tag{18}
\]
where \(v_i^*\) and \(\Delta x_i^*\) denote the \(i\)th elements of \(v^*\) and \(\Delta x^*\) defined as in (8) and (11), respectively.

The monotonicity of the interval quadratic programming is defined as the existence of \(\sigma_{i,j}^{(v)} \in \{-1, 1\}\) and \(\sigma_{i,j}^{(dx)} \in \{-1, 1\}\) such that (18), which means that the signs of \(\partial v_i^*/\partial d_j\) and \(\partial \Delta x_i^*/\partial d_j\) are invariant. Note that, if the interval quadratic programming is monotone with respect to \(d\), then the upper and lower limits of \(v_i^*\) are exactly given as
\[
\bar{v}_i^* = v_i^*(\underline{d}^{(i)}), \quad \underline{v}_i^* = v_i^*(\overline{d}^{(i)}) \text{ where the } j\text{th elements of } \underline{d}^{(i)} \in \mathbb{R}^n \text{ and } \overline{d}^{(i)} \in \mathbb{R}^n \text{ are defined by}
\[
\underline{d}_j^{(i)} := \sigma_{i,j}^{(v)} \max \left\{ \sigma_{i,j}^{(v)} d_j, \sigma_{i,j}^{(v)} \overline{d}_j \right\},
\overline{d}_j^{(i)} := \sigma_{i,j}^{(v)} \min \left\{ \sigma_{i,j}^{(v)} d_j, \sigma_{i,j}^{(v)} \underline{d}_j \right\}.
\]
This fact implies that the solutions \(v^*\) with a finite number of \(d\) exactly give the upper and lower limits of \(v^*\). Obviously, we can obtain the upper and lower limits of \(\Delta x^*\) in the same manner.

As for the monotonicity of the interval quadratic programming in Problem 1, we can prove the following theorem:

**Theorem 1.** Consider the interval quadratic programming in Problem 1. If
\[
\sigma_{i,j}^{(v)} = \begin{cases} 1, & i = j, \\ -1, & i \neq j, \end{cases} \quad \sigma_{i,j}^{(dx)} = -1, \tag{19}
\]
then (18) follows for any \(i \in \{1, \ldots, n\}\) and \(j \in \{1, \ldots, n\}\).

This theorem shows that the interval quadratic programming in Problem 1 possesses the monotonicity characterized by (19). Note that the upper and lower limits of \(\Delta x^*\) can be obtained by solving the quadratic programming with \(\underline{d}\) and \(\overline{d}\). Consequently, the solution of Problem 1 is exactly obtained by solving the quadratic programming with 2\(n + 2\) kinds of \(d\).

Recall that, in Problem 1, the equality constraint in (14) is given by the nominal PV/demand prediction \(d\) in (13). If the equality constraint also depends on \(d\), the interval quadratic programming does not possess the monotonicity in general. Thus, in this case, we cannot obtain the upper and lower limits in (16) and (17) by a finite number of calculation; recall the last of Section 2.2. The efficiency of analyses based on this monotonicity is demonstrated by numerical simulations in Section 4.

4. NUMERICAL SIMULATION

4.1 Prediction of PV Generation and Demand

In this section, we show the efficiency of the proposed method to plan power generation and battery charge pattern. We consider the supply-demand balance in Tokyo area having 19 million demanders, where five million demanders have PV generators and three million demanders have storage batteries.
by the mark of * on the solid line with triangles in Fig. 2(a). By adding the temporal sequences of the PV generation prediction intervals and demand prediction intervals, we obtain the sequence of the PV/demand prediction interval shown in Fig. 2(c), where the central solid line with circles represents the nominal PV/demand prediction \(d\) in (13). Note that, in this figure, we have already subtracted 20 GW to be covered by basis generators, such as nuclear plants and so forth.

4.2 Evaluation Indices for Power Generation Planning

We introduce some quantitative evaluation indices for power generation planning. We suppose that a desirable plan of power generation accomplishes the following conditions as much as possible:

(i) The maximum number of operated generators is minimized.
(ii) The change rate for the number of operated generators is minimized.
(iii) The number of generators to tolerate the prediction uncertainty is minimized.

From the viewpoints of (i), (ii) and (iii), we evaluate a resultant power generation plan.

The optimal solution for the nominal PV/demand prediction, is expressed as \(v^* = v^*(d)\) where \(v^*\) is defined as in (8). We refer to \(v^*\) as the nominal power generation plan. The upper and lower limits of the optimal power generation plan are defined as in (16).

Let us mathematically describe the evaluation indices (i), (ii) and (iii) as

\[
\begin{align*}
W_1 &:= \max_{i \in \{1, \ldots, 48\}} v_{i}^* , \quad W_2 := \sum_{i=1}^{47} |v_{i+1}^* - v_i^*| \\
W_3 &:= \sum_{i=1}^{48} \frac{v_i^* - v^*}{v_i^*}
\end{align*}
\]

(20)

where \(v_i^*\) denotes the \(i\)th element of \(v^*\). If the value of \(W_1\) is small, the maximum number of operated generators is small. Similarly, if the value of \(W_2\) is small, we do not require high readiness of generators, and if the value of \(W_3\) is small, the number of generators to tolerate the prediction uncertainty is small.

4.3 Planing of Power Generation and Battery Charge Pattern

The simulation results are shown in Figs. 3(a), (b) and (c) where we use the PV/demand prediction defined in Section 4.1. In these figures, we show the results in the cases where we vary the upper and lower limits of the GW capacity of batteries as Case 1: \(\pm 2.5\) GW, Case 2: \(\pm 4.0\) GW and Case 3: \(\pm 5.5\) GW. In each figure, the first subfigure shows the resultant plans of power generation \(v^*\) and battery charge pattern \(\Delta x^*\), defined as in (8) and (11), and the second one shows the corresponding temporal sequence of battery energy \(x\), defined as in (12). The solid lines with circles represent the nominal power generation plan and the corresponding nominal battery charge plan and the temporal sequence of battery energy. The thick solid lines show the upper and lower limits of all possible solutions for any \(d \in [d, \bar{d}]\). These lines can be obtained in a few seconds by solving the quadratic programming with \(2n + 2\) kinds of \(d\) because the problem in this simulation possesses the monotonicity. The indication by the color density is explained in Section 4.4.

We can see from each first subfigure that, as the GW capacity of batteries becomes larger, the difference between the upper and lower limits of the generated power becomes smaller. Furthermore, as shown in each second subfigure, the nominal temporal sequence of the battery energy at the termination time returns to its initial value. This is ensured by setting \(x_d\) in (6) to zero for the equality constraint in (14). Moreover, the difference between the upper and lower limits of the battery energy increase with the lapse of time. This is because the battery energy is defined by the temporal integration of charged power.

Next, in Table 1, we show the values of the evaluation indices in (20) for Case 1, Case 2 and Case 3. In this table, we normalize the the evaluation index values so that \(W_1 = W_2 = W_3 = 1\) holds if no storage batteries are installed. In this case, the temporal sequence of generated power must be identical to that of the PV/demand prediction; namely, it follows that \(d = v^*, \; \bar{d} = \bar{v}^*, \; \underline{d} = \underline{v}^*\). We can see from Table 1 that every evaluation index decreases in the order from Case 1 to Case 3. This fact indicates that batteries possessing a larger GW capacity is more effective to provide a desirable power generation plan, from a viewpoint of (i), (ii) and (iii).

Note that the value of \(W_3\) in Case 1 is greater than one. This means that we require a regulating capacity that is larger than one in the case where no storage batteries are installed. This may be caused by the fact \(\sigma_i^{(v)} \neq \sigma_i^{(\Delta x)}\) shown in Theorem 1, which means that a set of demand corresponding to the upper and lower limits of the optimal power generation plan is not identical to that of the optimal battery charge pattern plan. In conclusion, we see that the storage batteries with larger GW capacity are required to tolerate a larger amount of uncertainty.

4.4 Discussion on Total Battery Capacity

We investigate a total battery capacity that is required to tolerate prediction uncertainty. The temporal sequence of battery energy is shown in the each second subfigure of Figs. 3(a), (b) and (c). We can see from these figures that the differences between the peak values of the upper and lower limits are 108 GWh, 113 GWh and 119 GWh, respectively. To tolerate the worst case, we need to have storage batteries whose total GWh capacity is about 120 GWh; see Case 3.

Next, to investigate an occurrence rate of the worst case, we calculate the optimal plans of power generation and battery charge pattern for 10000 PV/demand predictions randomly chosen from \([d, \bar{d}]\). In Figs. 3(a), (b) and (c), the

<table>
<thead>
<tr>
<th>Case</th>
<th>(W_1)</th>
<th>(W_2)</th>
<th>(W_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.86</td>
<td>0.47</td>
<td>1.41</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.78</td>
<td>0.26</td>
<td>0.72</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.70</td>
<td>0.07</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Table 1. Normalized Values of Indices for Cloudy Day.
5. CONCLUSION

In this paper, we have formulated a problem to plan power generation and battery charge pattern as an interval quadratic programming. Furthermore, we have proposed a solution method to this problem. The solution of the interval quadratic programming can be regarded as the regulating capacity that can tolerate any uncertain demand prediction. Moreover, we have provided a number of simulation results to verify the efficiency of the proposed method for power generation planning. By these numerical simulations, we have found the following facts:

- Storage batteries having larger GW capacity are more effective to minimize the regulating capacity.
- If prediction uncertainty is relatively large compared with the GW capacity of batteries, we require regulating capacity that is larger than one in the case where no storage batteries are installed.

A generalization to multiple generators as well as a consideration of PV generation surplus are currently under investigation.

REFERENCES


