A Distributed Scheme for Power Profile Market Clearing under High Battery Penetration ⋆

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Abstract: In this paper, we formulate a problem of power profile market clearing and develop a distributed market clearing scheme with explicit consideration of high battery penetration. The power profile market is a multiperiod electricity market in which each aggregator aims at making the highest profit by transacting a power profile, i.e., a time sequence of energy amounts at several time slots, that is generated by dispatchable power generation as well as the charge and discharge of batteries. It is theoretically shown that the clearing price profile during the time period of interest tends to level off in the high penetration of batteries. This finding enables to develop a distributed market clearing scheme that is implemented as a bidding strategy for the total energy amount during the period followed by a distributed iterative algorithm for profile imbalance minimization. Numerical simulations demonstrate the price leveling-off led by high battery penetration and the efficiency of the proposed distributed scheme.

Keywords: Multiperiod electricity markets, Energy storage, Power profile balancing, Bidding strategy, Convex analysis.

1. INTRODUCTION

The development of a smart grid has been recognized as one of key issues in addressing environmental and social concerns, such as the sustainability of energy resources and the efficiency of energy management [Annaswamy and Amin (2013)]. In particular, towards effective integration of dispatchable and renewable power generation, the potential of energy storage has been attracting international attention in smart grid community. Actually, energy storage techniques can be expected as a fundamental tool for load shifting as well as reducing the fluctuation of renewable energy.

The penetration of energy storage is generally supposed to be spatially distributed due to the limitation of installation capability. Examples of distributed energy storage include electric vehicles, home energy storage systems, batteries in electric devices, and so forth. Even though the impact of these individual materials and components on the grid may be tiny, the aggregation of them has high potential to serve for supply-demand balancing in power system operation. This implies that an aggregator, a manager of available energy resources including energy storage, can be a strong stakeholder in an electricity market.

With this background, we formulate an electricity market mechanism with explicit consideration of battery penetration, which is referred to as a *power profile market mechanism*. A power profile market is a multiperiod electricity market in which each aggregator aims at making the highest profit by transacting the time sequence of energy amounts at several time slots. This energy amount sequence, formulated as a vector having the dimension compatible with the number of time slots, is referred to as a *power profile*. Each aggregator generates a marketable power profile by aggregating available energy resources, such as dispatchable and renewable power generation and the charge and discharge of distributed batteries. As shown in Section 4.3 of [Annaswamy and Amin (2013)], such a multiperiod market is indispensable for making use of the *power shiftability* of batteries and flexible loads. This is because their utility or cost function is not an additively decomposable function of period-specific power consumption and generation.

To establish a mathematically rigorous formulation of the power profile market mechanism, we first derive a regulation cost function of marketable power profiles,

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consisting of load, dispatchable power generation, and battery charge and discharge power profiles. Then, we show that the profile regulation cost function is necessarily convex provided that the aggregator adopts the optimal strategy for managing dispatchable power generators and batteries, whose cost functions are assumed to be both convex. This clarification enables to formulate the power profile market clearing problem as a convex program.

Furthermore, we develop a distributed solution scheme to the power profile market clearing problem, which can be implemented as an indirect communication among aggregators through an independent system operator (ISO). The market clearing scheme is developed by theoretically showing that the clearing price profile, i.e., the multiperiod clearing price vector of the power profile market, tends to level off in high penetration of batteries. Numerical simulations in this paper demonstrate the price leveling-off as well as the efficiency of the proposed distributed scheme, which consists of a bidding strategy for the total energy amount during the period and a distributed iterative algorithm for profile imbalance minimization.

Finally, references related to electricity markets are discussed. As market clearing strategies, a number of bidding and dynamic pricing methods have been developed in different settings; see [Hansen et al. (2015); He et al. (2015); Liu et al. (2016); Shiltz et al. (2016)] and references therein. However, these existing methods are not directly applicable to the power profile market clearing problem. This is due to the fact that a transacted power profile is a high-dimensional vector and the cost function of power profile regulation is not strictly convex because of the power shiftability of batteries; see Section 2.3 for details. Furthermore, even though the efficiency and significance of their methods are demonstrated numerically, the structures and properties of market mechanisms are not theoretically investigated. In contrast to this, by utilizing tools from convex analysis theory, we clarify a particular impact of high battery penetration on multiperiod market mechanisms on the basis of a simple but meaningful mathematical formulation.

The remainder of this paper is structured as follows. In Section 2, we first formulate the power profile market clearing problem, and then discuss the difficulties in addressing it. Next, in Section 3, we develop a distributed market clearing scheme while clarifying that the clearing price profile tends to level off in high battery penetration. Numerical simulations are provided in Section 4 and concluding remarks are provided in Section 5. All mathematical proofs are shown in Appendix.

Notation: We denote the set of real values by $\mathbb{R}$, the set of nonnegative real values by $\mathbb{R}_+$, the image of a matrix M by $\operatorname{im} M$, the all-ones vector by $\mathbf{1}$, the orthogonal projection of a vector v onto a subspace $\mathcal{V}$ by $\operatorname{proj}_{\mathcal{V}}(v)$, and the direct product of sets $S_1, \ldots, S_n$ by

$$S_1 \times \cdots \times S_n = \prod_{i \in \{1,\ldots,n\}} S_i.$$

A function $F : \mathbb{R}^n \to \mathbb{R}$ is said to be convex if

$$F\big((1-\lambda)x + \lambda x'\big) \le (1-\lambda)F(x) + \lambda F(x') \qquad (1)$$

for all $\lambda \in (0,1)$ and for every pair of x and x' in the domain such that the value of F is finite. In particular, F is said to be strictly convex if (1) holds with the strict inequality unless $x = x'$.

2. FORMULATION OF POWER PROFILE MARKETS

2.1 Aggregator Models

In this subsection, we give a model of aggregators, each of whom transacts a power profile, i.e., the time sequence of energy amounts at several time slots. Let $\mathcal{A}$ denote the index set of aggregators and let n denote the number of time slots during the period of interest. The power profile equation of the αth aggregator can be described as

$$x_\alpha = g_\alpha - l_\alpha + \eta_\alpha^{\mathrm{out}} \delta_\alpha^{\mathrm{out}} - \tfrac{1}{\eta_\alpha^{\mathrm{in}}} \delta_\alpha^{\mathrm{in}}, \quad \alpha \in \mathcal{A} \qquad (2)$$

where $x_\alpha \in \mathbb{R}^n$ denotes the resultant power profile to the grid, $g_\alpha \in \mathbb{R}_+^n$ denotes the power generation profile of dispatchable generators, $l_\alpha \in \mathbb{R}_+^n$ denotes the load profile, and $\delta_\alpha^{\mathrm{in}} \in \mathbb{R}_+^n$ and $\delta_\alpha^{\mathrm{out}} \in \mathbb{R}_+^n$ denote the battery charge and discharge power profiles. The positive constants $\eta_\alpha^{\mathrm{in}}$ and $\eta_\alpha^{\mathrm{out}}$ denote the charge and discharge efficiency, respectively, each of which takes a value in $(0,1]$. Note that the sign of x_α is positive for outflow direction to the grid.

In the following, we suppose that the load profile l_α is fixed as a constant vector, whereas the dispatchable power generation profile g_α as well as the battery charge and discharge power profiles $\delta_\alpha^{\mathrm{in}}$ and $\delta_\alpha^{\mathrm{out}}$ are decision variables. To realize a desired power profile x_α, each aggregator determines g_α and $\delta_\alpha := (\delta_\alpha^{\mathrm{in}}, \delta_\alpha^{\mathrm{out}})$ as complying with the constraints of

$$g_\alpha \in \mathcal{G}_\alpha, \quad \delta_\alpha \in \mathcal{D}_\alpha, \qquad (3)$$

where $\mathcal{G}_\alpha$ and $\mathcal{D}_\alpha$ denote some connected spaces including the origin. The left condition in (3) is given to represent the upper and lower bounds for the dispatchable generator outputs, whereas the right is given to represent the limitation of inverter and battery capacities.

With respect to each power profile x_α, we denote the feasible subspace of the dispatchable power generation and the battery charge and discharge profiles as

$$\mathcal{F}_\alpha(x_\alpha) := \big\{ (g_\alpha, \delta_\alpha) \in \mathcal{G}_\alpha \times \mathcal{D}_\alpha : (2) \text{ is satisfied} \big\}, \qquad (4)$$

and denote the set of realizable power profiles as

$$\mathcal{X}_\alpha := \big\{ x_\alpha \in \mathbb{R}^n : \mathcal{F}_\alpha(x_\alpha) \neq \emptyset \big\}. \qquad (5)$$

Furthermore, we denote the generation cost function of dispatchable generators and the battery usage cost function as

$$G_\alpha : \mathcal{G}_\alpha \to \mathbb{R}_+, \quad D_\alpha : \mathcal{D}_\alpha \to \mathbb{R}_+. \qquad (6)$$

On the basis of this formulation, we define a cost function with respect to power profile regulation as follows.

Lemma 1. In the notation above, if the generation cost function G_α and the battery usage cost function D_α are convex on convex domains $\mathcal{G}_\alpha$ and $\mathcal{D}_\alpha$, then the profile regulation cost function defined by

$$F_\alpha(x_\alpha) := \min_{(g_\alpha, \delta_\alpha) \in \mathcal{F}_\alpha(x_\alpha)} \Big\{ G_\alpha(g_\alpha) + D_\alpha(\delta_\alpha) \Big\} \qquad (7)$$

is convex on the convex domain $\mathcal{X}_\alpha$.

The value of $F_\alpha(x_\alpha)$ in (7) represents the minimum cost to realize a power profile x_α. Lemma 1 shows that the profile regulation cost function turns out to be convex with respect to generated power profiles, provided that the aggregator adopts the optimal strategy for the determination of the dispatchable power generation profile g_α and the battery charge and discharge power profile δ_α. In the rest of this paper, we assume the convexity of G_α and D_α, which implies that every F_α is convex.

2.2 Power Profile Market Clearing Problem

In this subsection, we formulate a market clearing problem with respect to power profile transaction based on the aggregator model in Section 2.1. Let $\lambda \in \mathbb{R}^n$ denote the price profile with respect to power profile transaction. Note that this vector corresponds to the set of prices at all time slots during the period of interest. Then, the profit function for selling surplus power (or for buying shortage power) during the period is defined as

$$J_\alpha(x_\alpha; \lambda) := \lambda^\mathsf{T} x_\alpha - F_\alpha(x_\alpha), \tag{8}$$

where $\lambda^\mathsf{T} x_\alpha$ represents the total income by selling the power profile during the time period. On the basis of this profit function, each aggregator can determine a bid function with respect to power profile transaction given as

$$\boldsymbol{x}_\alpha(\lambda) := \left\{ x_\alpha \in \mathcal{X}_\alpha : J_\alpha(x_\alpha; \lambda) \geq J_\alpha(x'_\alpha; \lambda),\ \forall x'_\alpha \in \mathcal{X}_\alpha \right\}, \tag{9}$$

which corresponds to the set of x_α that attain the maximum of the profit function $J_\alpha(x_\alpha; \lambda)$ with a fixed price profile λ. The bid function is a set valued function, i.e., it is mathematically referred to as a correspondence

$$\boldsymbol{x}_\alpha : \mathbb{R}^n \to \mathbb{R}^n. \tag{10}$$

It should be noted that the graph of $\boldsymbol{x}_\alpha$ is depicted as a one-dimensional curve for $n = 1$, whereas it is depicted as a higher-dimensional hyperplane for a general dimension n.

In the rest of this paper, for simplicity of notation, we denote the direct product of $\boldsymbol{x}_\alpha$ as

$$\boldsymbol{x}_\mathcal{A}(\lambda) := \textstyle\prod_{\alpha \in \mathcal{A}} \boldsymbol{x}_\alpha(\lambda).$$

Furthermore, the tuple of a symbol indexed by $\alpha \in \mathcal{A}$ is denoted by the corresponding symbol with the subscript of $\mathcal{A}$, for example

$$x_\mathcal{A} := (x_\alpha)_{\alpha \in \mathcal{A}}.$$

The following lemma shows the existence of a clearing price profile, determined via the aggregation of power profile bid functions.

Lemma 2. Consider a set of aggregators in Section 2.1. If at least one profile regulation cost function F_α is smooth, then there exists the unique clearing price profile, denoted as λ^*, such that the power profile balance

$$\exists x^*_\mathcal{A} \in \boldsymbol{x}_\mathcal{A}(\lambda^*) \ \text{ s.t. } \ \textstyle\sum_{\alpha \in \mathcal{A}} x^*_\alpha = 0 \tag{11}$$

is attained.

The problem of power profile market clearing is formulated as a problem of finding the clearing price profile and the tuple of clearing power profiles, denoted as λ^* and $x^*_\mathcal{A}$, respectively, that attain the power profile balance in (11). More formally, the power profile market clearing problem is summarized as follows.

Problem. Consider a set of aggregators in Section 2.1. Devise a market clearing scheme to realize the power profile balance in (11) such that the following requirements are satisfied.

- The clearing price profile λ^* is determined by the ISO without the information of the profile regulation cost functions of the aggregators.
- Each clearing power profile x^*_α is determined by an aggregator without the information of the profile regulation cost functions of the other aggregators.

2.3 Difficulties in Power Profile Market Clearing Problem

In this subsection, we overview the difficulties in addressing the power profile market clearing problem in Section 2.2, while reviewing several existing methods for market clearing. The problem of power profile market clearing is equivalent to finding a solution to the convex program of

$$\min_{x_\mathcal{A}} \textstyle\sum_{\alpha \in \mathcal{A}} F_\alpha(x_\alpha) \ \text{ s.t. } \ \sum_{\alpha \in \mathcal{A}} x_\alpha = 0, \tag{12}$$

whose Lagrange relaxation is given by

$$\max_\lambda \min_{x_\mathcal{A}} \textstyle\sum_{\alpha \in \mathcal{A}} \left\{ F_\alpha(x_\alpha) - \lambda^\mathsf{T} x_\alpha \right\}. \tag{13}$$

Note that the optimal Lagrange multiplier of (13) corresponds to the clearing price profile. This can be seen from the fact that the minimization in (13) is equivalent to the maximization of the profit function J_α in (8); thereby leading to the problem of finding $x^*_\mathcal{A}$ and λ^* such that (11) holds.

One may think that the ISO can easily find the clearing price profile λ^*, provided that each aggregator submits the graph of the bid function $\boldsymbol{x}_\alpha$ in (9) to the ISO. However, not only finding λ^* that solves the generalized equation in (11), but also plotting the graph of the bid function $\boldsymbol{x}_\alpha$ is not straightforward in multi-dimensional cases, i.e., $n \geq 2$. This is due to the fact that the graph of each element of $\boldsymbol{x}_\alpha$ is depicted as an n-dimensional hyperplane. This implies that plotting the graph of each element of $\boldsymbol{x}_\alpha$ requires griding the n-dimensional space of price profiles; thereby incurring considerably large computation loads. Thus, existing bidding strategies, such as in [Liu et al. (2016)], are not directly applicable to the power profile market clearing problem.

As another approach, dynamic pricing methods can be found in the literature [Hansen et al. (2015); Shiltz et al. (2016)]. This approach is mainly based on the dual ascent algorithm to solve the convex program in (12), or equivalently (13), namely

$$\begin{cases} x^{k+1}_\alpha = \arg\min\limits_{x_\alpha} F_\alpha(x_\alpha) - \lambda_k^\mathsf{T} x_\alpha, \quad \alpha \in \mathcal{A} \\ \lambda^{k+1} = \lambda^k + \gamma^k \sum_{\alpha \in \mathcal{A}} x^{k+1}_\alpha, \end{cases} \tag{14}$$

where γ^k denotes a step size to update the dual variable. Even though this algorithm can be implemented in a distributed manner, the update of each primal variable x^k_α assumes the strict convexity of the cost function F_α. In fact, the cost function of an aggregator with battery storage does not generally satisfy the assumption of the *strict* convexity; see Section 3.1 for details.

As seen above, neither a bidding strategy nor a dynamic pricing method is directly applicable. Thus, it is crucial to develop a novel distributed scheme for the power profile market clearing problem.

3. POWER PROFILE MARKET CLEARING SCHEME

3.1 Price Analysis under High Battery Penetration

In this subsection, we first analyze a property of the clearing price profile in a situation where a large amount of battery storage penetrates in aggregators. In particular, it will be found that the clearing price profile tends to level off in high battery penetration. Furthermore, the level-off price can be found via a bidding strategy with respect to the total amount of energy transaction during the period of interest.

For the price analysis below, we introduce a notion of directional derivatives. The one-sided directional derivative of F_α at x_α with respect to v is defined as

$$F'_\alpha(x_\alpha; v) = \lim_{\epsilon \downarrow 0} \frac{F(x_\alpha + \epsilon v) - F(x_\alpha)}{\epsilon}$$

if it exists. In particular, the one-sided directional derivative is said to be two-sided if $F'_\alpha(x_\alpha; -v)$ exists and

$$F'_\alpha(x_\alpha; -v) = -F'_\alpha(x_\alpha; v).$$

Given this notation, we define the following terminology.

Definition. Consider an aggregator in Section 2.1, and let

$$(i,j) \in \{1,\ldots,n\} \times \{1,\ldots,n\}$$

denote a time point pair. Then, a power profile x_α is said to be (i,j)-*shiftable* if the directional derivative of the profile regulation cost function F_α at x_α with respect to $e_i - e_j$ is two-sided and zero, where e_i denotes the ith canonical unit vector.

An aggregator with battery storage is generally endowed with this notion of power profile shiftability. This is because the amounts of battery charge and discharge power at some time slots can be shifted without changing the total amount of charge and discharge energy during the time period of interest. Note that F_α is not strictly convex whenever it is (i,j)-shiftable. This implies that the profile regulation cost function of an aggregator with battery storage does not satisfy the assumption of strict convexity in general.

The following lemma shows that the power profile shiftability can lead to the leveling-off of clearing price profiles.

Lemma 3. Consider a set of aggregators in Section 2.1, and let λ^* denote the clearing price profile in (11). Then, there exists at least one aggregator such that a power profile $x_\alpha \in \boldsymbol{x}_\alpha(\lambda^*)$ is (i,j)-shiftable if and only if

$$\lambda_i^* = \lambda_j^*, \tag{15}$$

where λ_i^* denotes the ith element of λ^*.

Lemma 3 shows that the shiftability of an optimal power profile $x_\alpha \in \boldsymbol{x}_\alpha(\lambda^*)$ that maximizes the profit function $J(x_\alpha; \lambda^*)$ in (8) leads to the leveling-off of the clearing price profile λ^*. Note that the degree of power shiftability tends to increase as the capacities of batteries increase. Therefore, if a sufficiently large amount of battery storage penetrates in aggregators, we can expect that the prices at all time slots are close, i.e., λ^* is close to $\lambda_{\mathrm{e}}^* \mathbf{1}$ for some scalar λ_{e}^*. In what follows, focusing on this situation of high battery penetration, we develop a distributed scheme for profile market clearing

The following theorem shows that the value of λ_{e}^*, called the clearing energy price, can be determined by a bidding strategy with respect to the total amount of energy transaction during the period of interest, where we use the energy bid function defined by

$$\boldsymbol{e}_\alpha(\lambda_{\mathrm{e}}) := \mathbf{1}^\mathsf{T} \boldsymbol{x}_\alpha(\lambda_{\mathrm{e}}\mathbf{1}), \tag{16}$$

whose direct product is denoted as

$$\boldsymbol{e}_{\mathcal{A}}(\lambda_{\mathrm{e}}) := \textstyle\prod_{\alpha \in \mathcal{A}} \boldsymbol{e}_\alpha(\lambda_{\mathrm{e}}).$$

Theorem 1. Consider a set of aggregators in Section 2.1. If at least one profile regulation cost function F_α is smooth, then there exists the unique clearing energy price, denoted as λ_{e}^*, such that the energy balance

$$\exists e_{\mathcal{A}}^* \in \boldsymbol{e}_{\mathcal{A}}(\lambda_{\mathrm{e}}^*) \ \text{ s.t. } \ \textstyle\sum_{\alpha \in \mathcal{A}} e_\alpha^* = 0 \tag{17}$$

is attained. Furthermore, assume that

$$\lambda^* \in \operatorname{im} \mathbf{1} \tag{18}$$

for the clearing price profile λ^* in (11), or equivalently, assume that there exists at least one aggregator such that a power profile $x_\alpha \in \boldsymbol{x}_\alpha(\lambda^*)$ is (i,j)-shiftable for every time point pair (i,j). Then, λ^* coincides with $\lambda_{\mathrm{e}}^* \mathbf{1}$.

Theorem 1 provides a bidding strategy for determining the clearing energy price λ_{e}^*, which can be expected to approximate the clearing price profile as $\lambda^* \simeq \lambda_{\mathrm{e}}^* \mathbf{1}$ especially in high battery penetration. It should be noted that, in a manner similar to existing bidding strategies [Liu et al. (2016)], the ISO can easily find the value of λ_{e}^* in (17) by aggregating the graphs of energy bid functions submitted from individual aggregators. The graph of $\boldsymbol{e}_\alpha$ in (16) is reasonably plottable by each aggregator because it is just a one-dimensional curve, namely

$$\boldsymbol{e}_\alpha : \mathbb{R} \to \mathbb{R},$$

as opposed to a high-dimensional hyperplane of $\boldsymbol{x}_\alpha$ in (10).

3.2 Distributed Scheme for Imbalance Minimization

In this subsection, we develop a distributed scheme to find a tuple of power profiles that minimizes the amount of power profile imbalance. The alternating direction method of multipliers (ADMM) [Boyd et al. (2011)] provides a distributed iterative scheme as follows.

Theorem 2. Consider a set of aggregators in Section 2.1. Let λ be a price profile, and consider the iterative algorithm given as

$$x_\alpha^{k+1} = \operatorname{proj}_{\boldsymbol{x}_\alpha(\lambda)} \left(x_\alpha^k - \Delta_\alpha^k \right), \quad \alpha \in \mathcal{A} \tag{19}$$

where the update with respect to the index α is performed in ascending order and the kth-step power profile imbalance Δ_α^k is defined as

$$\Delta_\alpha^k := \textstyle\sum_{i \le \alpha - 1} x_i^{k+1} + \sum_{i \ge \alpha} x_i^k. \tag{20}$$

Then, for any initial value $x_{\mathcal{A}}^0 \in \boldsymbol{x}_{\mathcal{A}}(\lambda)$, it follows that

$$x_{\mathcal{A}}^\infty := \lim_{k \to \infty} x_{\mathcal{A}}^k$$

is a solution to the convex program of

$$\min_{x_{\mathcal{A}} \in \boldsymbol{x}_{\mathcal{A}}(\lambda)} \tfrac{1}{2} \left\| \textstyle\sum_{\alpha \in \mathcal{A}} x_\alpha \right\|^2. \tag{21}$$

In particular, if λ is given as the clearing price profile λ^* in (11), then

$$\textstyle\sum_{\alpha \in \mathcal{A}} x_\alpha^\infty = 0, \tag{22}$$

which implies that $x_{\mathcal{A}}^\infty$ coincides with $x_{\mathcal{A}}^*$ in (11).

Theorem 2 provides a distributed scheme to solve the imbalance minimization problem in (21). Note that the update in (19) can be performed by individual aggregators, provided that the power profile imbalance Δ_α^k in (20) is computed and informed by the ISO. Thus, the distributed scheme can be regarded as a communication among the aggregators through the ISO.

Combining the theoretical results in Theorems 1 and 2, we develop a distributed scheme for power profile market clearing as follows.

(i) Each aggregator submits the graph of the energy bid function $\boldsymbol{e}_\alpha$ in (16) to the ISO.
(ii) The ISO determines the clearing energy price λ_{e}^* that attains the energy balance in (17).
(iii) Each aggregator selects an initial power profile x_α^0 in $\boldsymbol{x}_\alpha(\lambda_{\mathrm{e}}^*\mathbf{1})$ and submits it to the ISO.
(iv) Each power profile x_α^k is updated according to the distributed scheme in (19) with $\lambda = \lambda_{\mathrm{e}}^*\mathbf{1}$ that attains the minimization of the resultant power profile imbalance.

This power profile market clearing scheme can be expected to work well in high battery penetration. This is owing to the fact that the resultant power profile imbalance is expected to be small if the approximation error between λ^* and $\lambda_{\mathrm{e}}^*\mathbf{1}$ is small. In particular, the power profile market clearing problem in Section 2.2 is exactly solved if $\lambda^* = \lambda_{\mathrm{e}}^*\mathbf{1}$. The efficiency of this scheme will be numerically demonstrated in Section 4 below.

4. NUMERICAL SIMULATION

4.1 Simulation Setup

We consider a power profile market that consists of three aggregators. The period of power profile market clearing is supposed to be six hours and the amounts of transacted power are determined at every 30 minutes, i.e., the dimension of transacted power profiles is $n = 12$. We suppose that the first two aggregators manage five million residential consumers with energy storage, whereas the third aggregator manages a thermal generator, namely

$$x_\alpha = -l_\alpha + \eta_\alpha^{\mathrm{out}}\delta_\alpha^{\mathrm{out}} - \tfrac{1}{\eta_\alpha^{\mathrm{in}}}\delta_\alpha^{\mathrm{in}}, \quad \alpha \in \{1,2\}, \tag{23}$$

and $x_3 = g_3$. In our simulation, $\eta_\alpha^{\mathrm{out}} = \eta_\alpha^{\mathrm{in}} = 0.95$ for $\alpha \in \{1,2\}$ are given for the charge and discharge efficiency of batteries. The inverter and battery capacities, corresponding to $\mathcal{D}_\alpha$ in (3), are supposed to be 12 [kW] and 24 [kWh], respectively, for one residential consumer. The load profile l_α is given as a net load profile including consumption and photovoltaic power generation during the period of 6 [h] to 12 [h], for which we use the data measured at houses in Ohta city, Japan.

With $\mathcal{G}_3 = \mathbb{R}_+^{12}$ in (3), the fuel cost function of the thermal generator is given as

$$G_3(g_3) = 0.19 \times \|g_3\|^2 + 2.5 \times \mathbf{1}^{\mathsf{T}} g_3, \tag{24}$$

which corresponds to the oil thermal generator in [Masuta and Yokoyama (2012)]. On the other hand, the battery usage cost function D_α in (6) is supposed to be decomposed as

$$D_\alpha(\delta_\alpha) = E_\alpha(\delta_\alpha) - S_\alpha(\delta_\alpha), \quad \alpha \in \{1,2\} \tag{25}$$

where E_α represents a battery degradation cost function given as

$$E_\alpha(\delta_\alpha) = 10^{-5} \times \|\delta_\alpha^{\mathrm{out}}\|^2, \tag{26}$$

and S_α represents an assessment function with respect to the stored energy at the termination time of the period of interest.

Let s_α^0 denote the initial amount of stored energy, i.e., the state of charge, which is defined as the deviation from a neutral value. Then, the amount of stored energy deviation at the termination time can be represented as

$$s_\alpha(\delta_\alpha) := s_\alpha^0 + \mathbf{1}^{\mathsf{T}}(\delta_\alpha^{\mathrm{in}} - \delta_\alpha^{\mathrm{out}}). \tag{27}$$

In our simulation, we set $s_\alpha^0 = 0$ for $\alpha \in \{1,2\}$, whose unit is [GW0.5h]. It is reasonable to suppose that a higher level of terminal stored energy is more preferable than a lower level, and vice versa. To take into account this aspect, each aggregator is supposed to assess the value of terminal stored energy as

$$S_\alpha(\delta_\alpha) := \begin{cases} \hat{\lambda}_\alpha \eta_\alpha^{\mathrm{out}} s_\alpha(\delta_\alpha), & s_\alpha(\delta_\alpha) \geq 0, \\ \frac{\hat{\lambda}_\alpha}{\eta_\alpha^{\mathrm{in}}} s_\alpha(\delta_\alpha), & s_\alpha(\delta_\alpha) < 0, \end{cases} \tag{28}$$

where $\hat{\lambda}_\alpha \in \mathbb{R}_+$ denotes a constant electricity price offered, e.g., by an independent energy supplier having a steady contract with the corresponding aggregator. Comparing the energy price λ_{e} with $\hat{\lambda}_\alpha$, each aggregator determines own energy bid function $\boldsymbol{e}_\alpha$ in (16), i.e., the battery usage strategy for buying and selling energy in the six-hour period. In our simulation, we set $\hat{\lambda}_1 = 5.0$ and $\hat{\lambda}_2 = 4.0$, whose unit is [Yen/kW0.5h]. On the basis of the above setting, we obtain the profile regulation cost function F_α in (7) for each aggregator.

It should be remarked that D_α in (25) is convex because E_α in (26) is convex and S_α in (28) is concave. To show the concavity of S_α, i.e., the convexity of $-S_\alpha$, we define

$$h(s) := \begin{cases} \hat{\lambda}_\alpha \eta_\alpha^{\mathrm{out}} s, & s \geq 0, \\ \frac{\hat{\lambda}_\alpha}{\eta_\alpha^{\mathrm{in}}} s, & s < 0, \end{cases}$$

which is concave for any values of $\eta_\alpha^{\mathrm{out}}, \eta_\alpha^{\mathrm{in}} \in (0,1]$ and $\hat{\lambda}_\alpha \in \mathbb{R}_+$. Note that s_α in (27) is affine and

$$S_\alpha(\delta_\alpha) = h \circ s_\alpha(\delta_\alpha).$$

Thus, the convexity of $-S_\alpha$ is proven by the fact that the inverse image of a convex function under an affine mapping is a convex function; see Appendix for details. Note that D_α in (25) is convex but it is not strictly convex.

4.2 Results

In the following, varying the levels of battery penetration, we solve the power profile market clearing problem in Section 2.2. In particular, we consider the three cases where 20%, 40%, and 60% of residential consumers have the batteries. In Fig. 1, we plot the resultant clearing price profile $\lambda^* \in \mathbb{R}^{12}$ for each case, which is obtained by solving the convex program in (13). As theoretically supported in Lemma 3, the clearing price profile tends to level off as the battery penetration level becomes high.

In the following, without knowing the exact value of the resultant clearing price profiles in Fig. 1, we solve the power profile market clearing problem according to the distributed scheme developed in Section 3.2. The graphs of the resultant energy bid functions $\boldsymbol{e}_\alpha$ in (16) are shown in Fig. 2. From the total energy bid functions given as

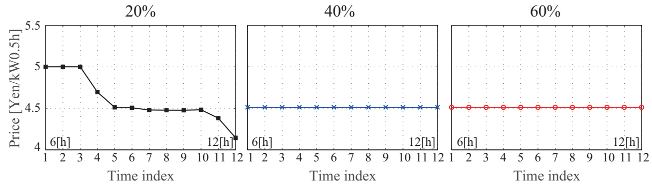

Fig. 1. The clearing price profiles in the cases of different battery penetration levels.

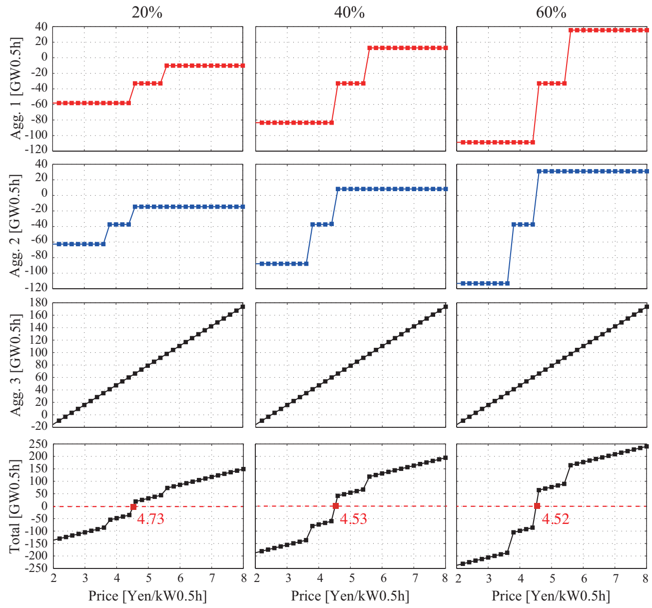

Fig. 2. The resultant energy bid functions in the cases of different battery penetration levels.

$\sum_{\alpha\in\mathcal{A}} \boldsymbol{e}_\alpha$, we find the clearing energy price $\lambda_{\rm e}^*$ in (17) as 4.75, 4.53 and 4.52 [Yen/kW0.5h] for the 20%, 40%, and 60% penetration levels, respectively. This procedure corresponds to Steps **(i)** and **(ii)** in our distributed scheme.

Next, we implement the iterative scheme in Theorem 2 that performs profile imbalance minimization. This procedure corresponds to Steps **(iii)** and **(iv)**. The norms of resultant profile imbalance are plotted in Fig 3, where the horizontal axis corresponds to the number of iteration steps. From this figure, we see that some amount of profile imbalance remains in the 20% penetration level, whereas almost no amount of imbalance remains in the 40% and 60% penetration levels. This is compatible with the fact shown in Theorem 2 that the clearing price profile in the 20% penetration level does not completely level off whereas those in the 40% and 60% penetration levels clearly level off; see Fig. 1. Finally, the resultant power profiles of the aggregators are plotted in Fig 4. From this figure, we see that our distributed scheme works well especially in high battery penetration.

5. CONCLUDING REMARKS

In this paper, we have formulated a power profile market clearing problem and develop a distributed market clearing scheme with explicit consideration of high battery penetration. It has been theoretically shown that the clearing price profile during the time period of interest tends to level off in the high penetration of batteries. On the basis

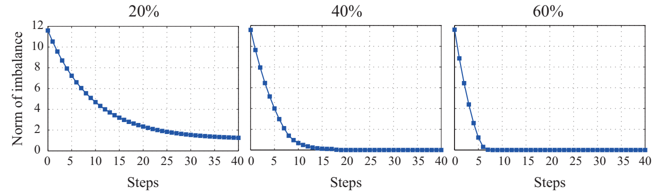

Fig. 3. The norms of profile imbalance in the cases of different battery penetration levels.

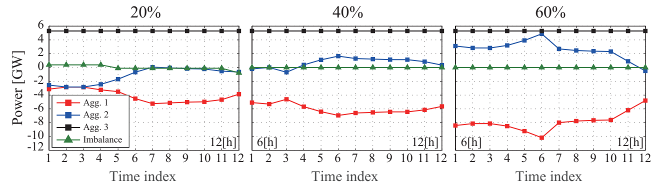

Fig. 4. The resultant power profiles in the cases of different battery penetration levels.

of this finding, we have developed a distributed market clearing scheme implemented as the combination of an energy bidding strategy and a distributed iterative algorithm for profile imbalance minimization.

The energy bidding strategy in this paper can be regarded as a long-period version of the current energy market. For example, an energy transaction is carried out at every 30 minutes in the Japanese electricity market. Our numerical simulation suggests that, provided that the level of battery penetration is sufficiently high, the energy balancing at every 30 minutes, i.e., power profile balancing, can be attained via the six-hour period energy transaction followed by a profile imbalance minimization scheme. The power profile market would be one of well-defined multiperiod electricity markets that can utilize and enhance the potential of energy storage.

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APPENDIX: MATHEMATICAL PROOFS

Preliminary Facts from Convex Analysis: We first overview a rich variety of facts shown in convex analysis theory [Boyd and Vandenberghe (2004); Rockafellar (1970)]. Let $F : \mathbb{R}^n \to \mathbb{R}$ be a convex function. For A denoting a linear mapping from $\mathbb{R}^n$ to $\mathbb{R}^m$, the function $\hat{F}_A$ defined by

$$\hat{F}(x) := F(Ax) \tag{29}$$

is convex on $\mathbb{R}^n$. Furthermore, the function $\check{F}_A$ defined by

$$\check{F}(y) := \inf_x F(x) \quad \text{s.t.} \quad Ax = y \tag{30}$$

is convex on $\mathbb{R}^m$. The function $\hat{F}$ is called the inverse image of F under A, while $\check{F}$ is called the image of F under A; see [Theorem 5.7, Rockafellar (1970)] for details.

The *conjugate* of F is defined by

$$\overline{F}(\lambda) := \sup_{x \in \mathrm{dom}\, F} \left\{ \lambda^\mathsf{T} x - F(x) \right\}, \tag{31}$$

where $\mathrm{dom} F$ denotes the effective domain of F. It is known that $\overline{F}$ is convex and the conjugate of $\overline{F}$ coincides with F as long as F is convex, namely

$$F(x) = \sup_{\lambda \in \mathrm{dom}\, \overline{F}} \left\{ x^\mathsf{T} \lambda - \overline{F}(\lambda) \right\}. \tag{32}$$

Furthermore, the strict convexity of F is equivalent to the smoothness of $\overline{F}$. In convex analysis theory, the transformation between (31) and (32) is called the Legendre-Fenchel transformation, and some of conjugate pairs can be found in closed forms.

The subdifferential of a convex function F is defined by

$$\partial F(x) := \{\lambda : F(x') \geq F(x) + \lambda^\mathsf{T}(x' - x),\ \forall x' \in \mathrm{dom}\, F\},$$

which is a multivalued mapping to a convex set. As shown in [Corollary 31.5.2, Rockafellar (1970)], it follows that

$$\partial F : \mathbb{R}^n \to \mathbb{R}^n$$

is a monotone increasing mapping, namely

$$\big(\partial F(x) - \partial F(x')\big)^\mathsf{T} (x - x') \geq 0, \quad \forall x, x' \in \mathrm{dom}\, F,$$

if F is convex. In a similar manner, it is shown to be strictly monotone if F is strictly convex [Theorem 2.1, Luc (1994)]. Furthermore, [Theorem 23.5, Rockafellar (1970)] shows that

$$x \in \partial \overline{F}(\lambda), \quad \lambda \in \partial F(x)$$

are equivalent. These correspond to the first derivative conditions for the suprema in (31) and (32).

Proof of Lemma 1: We see that x_α in (2) is given as an affine mapping of $(g_\alpha, \delta_\alpha)$. As shown in (30), the image of a convex function under an linear mapping, or equivalently affine mapping, is a convex function. Thus, the convexity of F_α in (7) is proven. □

Proof of Lemma 2: From the definition of the conjugate function, we notice that

$$\sup_{x_\alpha} J_\alpha(x_\alpha; \lambda) = \overline{F}_\alpha(\lambda)$$

for J_α in (8). Furthermore, the first derivative condition for the supremum, namely

$$\frac{\partial J_\alpha(x_\alpha; \lambda)}{\partial x_\alpha} = 0,$$

leads to $\lambda \in \partial F_\alpha(x_\alpha)$. Because $\lambda \in \partial F_\alpha(x_\alpha)$ if and only if $x_\alpha \in \partial \overline{F}_\alpha(\lambda)$, we see that $\boldsymbol{x}_\alpha$ in (9) is identical to $\partial \overline{F}_\alpha$. Thus, (11) is equivalent to the generalized equation of

$$0 \in \textstyle\sum_{\alpha \in \mathcal{A}} \partial \overline{F}_\alpha(\lambda^*). \tag{33}$$

Note that $\partial \overline{F}_\alpha$ is strictly monotone if $\overline{F}_\alpha$ is strictly convex. Furthermore, $\overline{F}_\alpha$ is strictly convex if and only if F_α is smooth. Thus, provided that at least one F_α is smooth, the sum of monotone mappings in (33) is strictly monotone. This proves the uniqueness of λ^*, which solves the generalized equation in (33) with respect to a strictly monotone mapping. □

Proof of Lemma 3: From the fact that $x_\alpha \in \partial \overline{F}_\alpha(\lambda^*)$, we have $\lambda^* \in \partial F_\alpha(x_\alpha)$. As shown in [Theorem 23.4, Rockafellar (1970)], it follows that

$$F'_\alpha(x_\alpha; v) = \sup_{\lambda \in \partial F_\alpha(x_\alpha)} v^\mathsf{T} \lambda.$$

Thus, (i, j)-shiftability of $x_\alpha \in \partial \overline{F}_\alpha(\lambda^*)$ implies that

$$0 = F'_\alpha(x_\alpha; e_i - e_j) \geq (e_i - e_j)^\mathsf{T} \lambda^*,$$
$$0 = F'_\alpha(x_\alpha; e_j - e_i) \geq (e_j - e_i)^\mathsf{T} \lambda^*,$$

or equivalently, $(e_i - e_j)^\mathsf{T} \lambda^* = 0$. This leads to (15). □

Proof of Theorem 1: Note that (17) is equivalent to

$$0 \in \textstyle\sum_{\alpha \in \mathcal{A}} \mathbf{1}^\mathsf{T} \partial \overline{F}_\alpha(\lambda_\mathrm{e}^* \mathbf{1}) = \partial \left(\sum_{\alpha \in \mathcal{A}} \overline{F}_\alpha(\lambda_\mathrm{e}^* \mathbf{1}) \right),$$

where the sum of $\overline{F}_\alpha(\lambda_\mathrm{e} \mathbf{1})$ is strictly convex with respect to λ_e, provided that at least one F_α is smooth. Thus, the uniqueness of λ_e^* is proven in a manner similar to the proof of Lemma 2.

Furthermore, (18) implies that $\lambda^* = \gamma \mathbf{1}$ holds for some scalar γ. Substituting this into (33) and multiplying it by $\mathbf{1}^\mathsf{T}$ from the left, we have

$$0 \in \textstyle\sum_{\alpha \in \mathcal{A}} \mathbf{1}^\mathsf{T} \partial \overline{F}_\alpha(\gamma \mathbf{1}),$$

which implies that γ solves the generalized equation. Because of the uniqueness of the solution, the scalar γ is identical to λ_e^*. This proves the claim. □

Proof of Theorem 2: We follow the arguments in [Boyd et al. (2011)] for deriving the distributed scheme in (19). A proof of convergence can be found in the reference or in [Rockafellar (1976)]. Notice that the convex program in (19) is equivalent to

$$\min_{x_\mathcal{A}, \epsilon} \tfrac{1}{2} \|\epsilon\|^2 + \textstyle\sum_{\alpha \in \mathcal{A}} I_{\boldsymbol{x}_\alpha(\lambda)} \quad \text{s.t.} \quad \sum_{\alpha \in \mathcal{A}} x_\alpha + \epsilon = 0$$

where $I_{\boldsymbol{x}_\alpha(\lambda)}$ denotes the indicator function of $\boldsymbol{x}_\alpha(\lambda)$. For this program, we consider the augmented Lagrangian in the scaled form as

$$\tfrac{1}{2} \|\epsilon\|^2 + \textstyle\sum_{\alpha \in \mathcal{A}} I_{\boldsymbol{x}_\alpha(\lambda)} + \tfrac{1}{2} \| \sum_{\alpha \in \mathcal{A}} x_\alpha + \epsilon + y \|^2$$

where y denotes the scaled dual variable. Then, the iteration of the ADMM is obtained as

$$
\begin{cases}
x_\alpha^{k+1} = \mathrm{proj}_{\boldsymbol{x}_\alpha(\lambda)}\left(-\left(\sum_{i<\alpha} x_i^{k+1} + \sum_{i>\alpha} x_i^k + \epsilon^k + y^k\right)\right) \\
\epsilon^{k+1} = -\frac{1}{2}\left(\sum_{\alpha\in\mathcal{A}} x_\alpha^k + y^k\right) \\
y^{k+1} = \frac{1}{2}\left(\sum_{\alpha\in\mathcal{A}} x_\alpha^k + y^k\right).
\end{cases}
$$

Note that ϵ^k and y^k in the update of x_α^k is redundant because of

$$
\epsilon^k + y^k = 0.
$$

Thus, this is reduced to (19). In particular, when $\lambda = \lambda^*$, there exists $x_{\mathcal{A}}^* \in \boldsymbol{x}_{\mathcal{A}}(\lambda^*)$ such that (11) holds. Thus, (22) follows. □